Answer on Question #60912 – Math – Calculus

Question

Find the Derivative of y=x^2e^(-3x)

Solution

It is known that

$$(e^x)' = e^x,$$

$$(x^n)' = nx^{n-1}, \quad \textbf{(1)}$$

Substituting $n = 2$ into (1) gives

$$(x^2)' = 2x.$$

Substituting $n = 1$ into (1) gives

$$x' = 1.$$

By a property of derivatives,

$$(k \cdot h(x))' = k \cdot h'(x), \quad \textbf{(2)}$$

where k is arbitrary real constant.

Substituting $k = -3$, $h(x) = x$ into (2) gives

$$(-3x)' = -3 \cdot x' = -3 \cdot 1 = -3.$$

Let $f(x) = e^x$, $g(x) = -3x$, then $f(g(x)) = e^{-3x}$.

The chain rule states that

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x).$$

Therefore,

$$(e^{-3x})' = (e^t)'|_{t=-3x} \cdot (-3x)' = e^t|_{t=-3x} \cdot (-3) = e^{-3x} \cdot (-3) = -3e^{-3x}.$$

Using the product rule $(uv)' = u'v + uv'$ for derivatives obtain

$$y' = (x^2e^{-3x})' = (x^2)'e^{-3x} + x^2(e^{-3x})' = 2x \cdot e^{-3x} + x^2 \cdot (-3e^{-3x}) = (2x - 3x^2)e^{-3x}$$

Answer: $y' = (2x - 3x^2)e^{-3x}$.