Answer on Question #60865 – Math – Trigonometry

Question

If sinθ **and cos**θ are the roots of equation ax² **-bx+c=0 then ?**

Solution

At first, factorize the polynomial ax² **-bx+c:**

$$ax^2 - bx + c = a(x - \sin\theta)(x - \cos\theta), \qquad a \neq 0$$

Simplifying the right-hand side obtain

$$ax^2 - bx + c = ax^2 - a(\sin\theta + \cos\theta)x + a \cdot \sin\theta\, cos\theta$$

Coefficients of the terms $x, x^0 = 1$ **in the left-hand and right-hand sides must be equal, hence by** Vieta's formulas obtain

$$\begin{cases} a \cdot \sin\theta\, cos\theta = c \\ a(\sin\theta + \cos\theta) = b \end{cases}$$

Let's derive *relation between* a, b *and* c*:*

$$b^2 = a^2(\sin\theta + \cos\theta)^2 = a^2(\sin^2\theta + \cos^2\theta + 2\sin\theta\, cos\theta) = a^2(1 + 2\sin\theta\, cos\theta) =$$

$$= a^2 + 2ac.$$

That is,

$$b^2 = a^2 + 2ac,$$

hence

$$\boldsymbol{c = \frac{b^2 - a^2}{2a}}.$$

Let's *find* θ*:*

$$\sin\theta + \cos\theta = \sqrt{2} \cdot \cos\left(\theta - \frac{\pi}{4}\right) = \frac{b}{a}$$

$$\cos\left(\theta - \frac{\pi}{4}\right) = \frac{b}{\sqrt{2}a}$$

$\theta - \frac{\pi}{4} = \pm\arccos\frac{b}{\sqrt{2}a} + 2\pi n,\ n \in \mathbb{Z}$, where arccos is the inverse of the cosine.

Finally, there are two possible families of solutions:

$$\boldsymbol{\begin{cases} \theta = \frac{\pi}{4} + \arccos\frac{b}{\sqrt{2}a} + 2\pi n, \\ \theta = \frac{\pi}{4} - \arccos\frac{b}{\sqrt{2}a} + 2\pi n, \end{cases}} \quad n \in \mathbb{Z}$$

Answer: $\theta = \frac{\pi}{4} + \arccos\frac{b}{\sqrt{2}a} + 2\pi n$ or $\theta = \frac{\pi}{4} - \arccos\frac{b}{\sqrt{2}a} + 2\pi n, n \in \mathbb{Z}$.