

Answer on Question #60859 – Math – Discrete Mathematics

Question

For the following decimal numbers, determine its binary representation. (Show full working)

- a. 200
- b. 300
- c. 85
- d. 77

Solution

Method 1

Divide downwards, dividing each new quotient by two and writing the remainders to the right. The quotient is the integer number under the long division symbol. Stop when the quotient is 0.

Starting with the bottom remainder, read the sequence of remainders upwards to the top and write out the binary representation.

a.	2) <u>200</u>	Remainder
	2) <u>100</u>	0
	2) <u>50</u>	0
	2) <u>25</u>	0
	2) <u>12</u>	1
	2) <u>6</u>	0
	2) <u>3</u>	0
	2) <u>1</u>	1
		1

So $200_{10} = 11001000_2$.

b.

	2) <u>300</u>	Remainder
	2) <u>150</u>	0
	2) <u>75</u>	0
	2) <u>37</u>	1
	2) <u>18</u>	1
	2) <u>9</u>	0
	2) <u>4</u>	1
	2) <u>2</u>	0
	2) <u>1</u>	0
		1

So $300_{10} = 100101100_2$.

c.

2) <u>85</u>	Remainder
2) <u>42</u>	1
2) <u>21</u>	0
2) <u>10</u>	1
2) <u>5</u>	0
2) <u>2</u>	1
2) <u>1</u>	0
	1

So $85_{10} = 1010101_2$.

d.

2) <u>77</u>	Remainder
2) <u>38</u>	1
2) <u>19</u>	0
2) <u>9</u>	1
2) <u>4</u>	1
2) <u>2</u>	0
2) <u>1</u>	0
	1

So $77_{10} = 1001101_2$.

Answer:

a. 11001000_2 ; b. 100101100_2 ; c. 1010101_2 ; d. 1001101_2 .

Method 2

Table 1

$2^8 = 256$	$2^7 = 128$	$2^6 = 64$
$2^5 = 32$	$2^4 = 16$	$2^3 = 8$
$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Represent the decimal as the sum of powers of 2 and apply formulae from Table 1. Finally we obtain the binary representation.

a.

$$\begin{aligned} 200 &= 128 + 64 + 8 = 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^3 = \\ &= 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 11001000_2. \end{aligned}$$

b.

$$300 = 256 + 32 + 8 + 4 = 1 \cdot 2^8 + 1 \cdot 2^5 + 1 \cdot 2^3 + 1 \cdot 2^2 =$$

$$= 1 \cdot 2^8 + 0 \cdot 2^7 + 0 \cdot 2^6 + 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 100101100_2.$$

c.

$$85 = 64 + 16 + 4 + 1 = 1 \cdot 2^6 + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^0 =$$

$$= 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1010101_2.$$

d.

$$77 = 64 + 8 + 4 + 1 = 1 \cdot 2^6 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^0 =$$

$$= 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 1001101_2.$$

Answer:

a. 11001000₂; **b.** 100101100₂; **c.** 1010101₂; **d.** 1001101₂.

Method 3

Table 2

$0_{10} = 0_{16} = 0000_2$	$1_{10} = 1_{16} = 0001_2$	$2_{10} = 2_{16} = 0010_2$	$3_{10} = 3_{16} = 0011_2$
$4_{10} = 4_{16} = 0100_2$	$5_{10} = 5_{16} = 0101_2$	$6_{10} = 6_{16} = 0110_2$	$7_{10} = 7_{16} = 0111_2$
$8_{10} = 8_{16} = 1000_2$	$9_{10} = 9_{16} = 1001_2$	$10_{10} = A_{16} = 1010_2$	$11_{10} = B_{16} = 1011_2$
$12_{10} = C_{16} = 1100_2$	$13_{10} = D_{16} = 1101_2$	$14_{10} = E_{16} = 1110_2$	$15_{10} = F_{16} = 1111_2$

Using division first we convert the decimal representation to hexadecimal one. Starting with the bottom remainder, read the sequence of remainders upwards to the top.

Then we shall convert the hexadecimal representation to binary one using formulae from Table 2.

The first digit in the binary representation should be non-zero.

a.

$$\begin{array}{r} 16 \overline{)200} \quad \text{Remainder} \\ \underline{16)12} \quad \quad 8 \\ \quad \quad \quad 12 \end{array}$$

$$\text{So } 200_{10} = 16 \cdot 12 + 8 = 12 \cdot 16^1 + 8 \cdot 16^0 = C8_{16} = 11001000_2.$$

b.

$$\begin{array}{r} 16 \overline{)300} \quad \text{Remainder} \\ \underline{16)18} \quad \quad 12 \\ \quad \quad \quad 16 \overline{)1} \quad \quad 2 \\ \quad \quad \quad \quad \quad 1 \end{array}$$

$$\text{So } 300_{10} = 1 \cdot 16^2 + 2 \cdot 16^1 + 12 \cdot 16^0 = 12C_{16} = 100101100_2.$$

c.

$$\begin{array}{r} 16 \overline{)85} \quad \text{Remainder} \\ \underline{16)5} \quad \quad 5 \\ \quad \quad \quad 5 \end{array}$$

$$\text{So } 85_{10} = 5 \cdot 16^1 + 5 \cdot 16^0 = 55_{16} = 1010101_2.$$

d. 16)77 Remainder
 16)4 13
 4

So $77_{10} = 4 \cdot 16^1 + 13 \cdot 16^0 = 4D_{16} = 1001101_2$.

Answer:

a. 11001000_2 ; **b.** 100101100_2 ; **c.** 1010101_2 ; **d.** 1001101_2 .