Answer on Question #60849 – Math – Calculus

Question

What is the coefficient of (x+1)^3 in the cubic Taylor polynomial about -1 for the function f(x)=e^x. The coefficient of (x+1)^3 is?

Solution

The cubic Taylor polynomial about -1 for the function $f(x) = e^x$ is

$$f(x) = f(-1) + \frac{1}{1!}f'(-1) \cdot (x+1) + \frac{1}{2!}f''(-1) \cdot (x+1)^2 + \frac{1}{3!}f'''(-1) \cdot (x+1)^3.$$

The coefficient of $(x+1)^3$ in the cubic Taylor polynomial about -1 for the function $f(x) = e^x$ is

$$\frac{1}{3!}f'''(-1) = \frac{1}{3!}(e^x)'''|_{x=-1} = \frac{1}{3!}e^x|_{x=-1} = \frac{e^{-1}}{6} = \frac{1}{6e}.$$

Answer: $\frac{1}{6e}$.