Answer on Question #60812 – Math – Calculus

Question

The Taylor series about 0 for the function

$f(x)=\sin(x)$ is $x - x^3/6 + x^5/120 – x^7/5040 + x^9/362880 + ...$

and the Taylor series about 0 for the function

$g(x)=e^x$ is $1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120 + ...$

What is the coefficient of x^3 in the series for $e^x \sin(x)$?

Solution

$e^x \sin(x) = (1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120 + ...)\cdot(x- x^3/6 + x^5/120 – x^7/5040 + x^9/362880+ ...)=$

$= x + x^2 + x^3/2 + x^4/6 - x^3/6 - x^4/6 + ... = x + x^2 + \left(\frac{1}{2} - \frac{1}{6}\right)x^3 + ... = x + x^2 + \frac{1}{3}x^3 + ...$

Answer: 1/3.