Question

A third plane can be found that passes through the line of intersection of two existing planes.?

a) Two planes are given by the equations $2x - 3y + z - 6 = 0$ and $-3x + 5y + 4z + 6 = 0$. Find the scalar equation of the plane that passes through the line of intersection of these two planes, and also passes through the point $A = (1, 3, -1)$.

b) Give the equations of two planes. Create a third plane that passes through the line of intersection of the original two and which is parallel to the z-axis. Explain your reasoning

Solution

a) At first, find equation of the line of intersection of two planes.

$$2x - 3y + z - 6 = -3x + 5y + 4z + 6 = 0$$

$$6x - 9y + 3z - 18 = 6x - 10y - 8z - 12$$

$$y + 11z - 6 = 0$$

Let $z = \lambda$. Then the general solution of equation

$$\begin{cases} 2x - 3y + z - 6 = 0 \\ y = 6 - 11z \end{cases}$$

is

$$\begin{cases} x = 12 - 17\lambda \\ y = 6 - 11\lambda \\ z = \lambda \end{cases}$$

Then find two points on this line, e.g. $B = (12, 6, 0)$ and $C = (-5, -5, 1)$. Let

$$\vec{AB} = (11, 3, 1)$$

$$\vec{AC} = (-6, -8, 2)$$

. The normal of the two vectors is:

$$n = \vec{AB} \times \vec{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 11 & 3 & 1 \\ -6 & -8 & 2 \end{vmatrix} =$$

$$\mathbf{i}\begin{vmatrix} 3 & 1 \\ -8 & 2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 11 & 1 \\ -6 & 2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 11 & 3 \\ -6 & -8 \end{vmatrix} = 14\mathbf{i} - 28\mathbf{j} - 70\mathbf{k}$$

The general equation of a plane is:

$$n \cdot (x - x_0, y - y_0, z - z_0) = 0$$

$$(14, -28, -70) \cdot (x - 1, y - 3, z + 1) = 0$$
$$14(x - 1) - 28(y - 3) - 70(z + 1) = 0$$
$$14x - 28y - 70z = 0$$

b) Equation of the plane which is parallel to the z-axis looks like $ax + by + d = 0$. The equation of the line of intersection and points B and C are the same as in previous task. So far as points $B = (12, 6, 0)$ and $C = (-5, -5, 1)$ belong to this plane:

$$\begin{cases} 12a + 6b + d = 0 \\ -5a - 5b + d = 0 \end{cases}$$

$$\begin{cases} 17a + 11b = 0 \\ -5a - 5b + d = 0 \end{cases}$$

Let $a = t$. The general solution of this equation is:

$$\begin{cases} a = t \\ b = -\frac{17t}{11} \\ d = -\frac{30t}{11} \end{cases}$$

The general equation of this plane is
$tx - \frac{17ty}{11} - \frac{30t}{11} = 0 \Rightarrow 11x - 17y - 30 = 0$

Answer

a) $14x - 28y - 70z = 0$

b) $11x - 17y - 30 = 0$