

Answer on Question #60733 – Math – Statistics and Probability

Question

A group of ten students from the course INTE 296 was approached by a famous scientist by the name of Dr. Wild E. Coyote with an experimental drug to aid in their retention of statistical procedures. The drug, called Memoraid™, was administered after the students completed a generic, comprehensive exam. Twenty-four hours later, the students were given a similar exam, and the test scores were compared. Based on the following table, is there enough evidence to conclude that the drug was effective in increasing their statistical ability? Use an alpha value of 0.05 and find the p-value.

Student	1	2	3	4	5	6	7	8	9	10
Before	75	62	70	70	55	59	60	64	72	65
After	74	65	68	74	62	62	60	67	75	58

Solution

Let X be the sample of the test scores before taking the drug, and Y be the sample of the test scores after taking the drug. The null hypothesis is $H_0: E(\bar{X}) = E(\bar{Y})$, and the alternative hypothesis is $H_a: E(\bar{X}) \neq E(\bar{Y})$. We complete the next table:

i	X_i	Y_i	$d_i = X_i - Y_i$
1	75	74	1
2	62	65	-3
3	70	68	2
4	70	74	-4
5	55	62	-7
6	59	62	-3
7	60	60	0
8	64	67	-3
9	72	75	-3
10	65	58	7

The volume of the samples is equal to $n = 10$; $\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i = -1.3$ is the mean, and

$s_d = \sqrt{\frac{\sum_{i=1}^n d_i^2 - \frac{(\sum_{i=1}^n d_i)^2}{n}}{n-1}} = 3.9$ is the corrected standard deviation of the sample $X - Y$. Then the next value has the distribution of Student with the $k = n - 1 = 9$ degrees of freedom:

$$T = \bar{d} \cdot \frac{\sqrt{n}}{s_d} = -1.05.$$

Then the p-value is equal to

$$p = \int_{-\infty}^{-1.05} f(x)dx + \int_{1.05}^{\infty} f(x)dx = 2 \int_{1.05}^{\infty} f(x)dx = 1 - 2 \int_0^{1.05} f(x)dx = 0.3211.$$

$f(x)$ is the density of the distribution of Student, $f(x) = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{k\pi}\Gamma\left(\frac{k}{2}\right)\left(1+\frac{x^2}{k}\right)^{\frac{k+1}{2}}}$.

In our case ($k = 9$)

$$f(x) = \frac{128}{105\pi\left(1+\frac{x^2}{9}\right)^5}.$$

Since $p = 0.3211 > \alpha = 0.05$ we have no reasons to reject the null hypothesis

$H_0: E(\bar{X}) = E(\bar{Y})$ and we must conclude that the drug was not effective in increasing of statistical ability of the students (at the level of significance of $\alpha = 0.05$).

Answer. There is not enough evidence to conclude that the drug was effective.