Answer on Question #60717 – Math – Other

Question

A marketing manager has five salesmen and five sales districts. Considering the capabilities of the salesmen and the nature of districts, the marketing manager estimates that sales per month (in hundred rupees) for each salesman in each district would be as follows:

		Districts				
		A	B	C	D	E
	1	32	38	40	28	40
	2	40	24	28	21	36
Salesman	3	41	27	33	30	37
	4	22	38	41	36	36
	5	29	33	40	35	39

Find the assignment of salesmen to districts that will result in maximum sales.

Solution

This is a maximization problem and has to be converted into a minimization problem by subtracting all the elements from the largest element of the sales table. Here the largest element is 41. Hence, the equivalent sales table for this problem would be obtained by reducing all the elements from 41 and rewriting it as follows:

$$\begin{pmatrix} 9 & 3 & 1 & 13 & 1 \\ 1 & 17 & 13 & 20 & 5 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 12 & 8 & 1 & 6 & 2 \end{pmatrix}$$

Step 1. Subtract the smallest element of each row from every element of the corresponding row, we get

$$\begin{pmatrix} 8 & 2 & 0 & 12 & 0 \\ 0 & 16 & 12 & 19 & 4 \\ 0 & 14 & 8 & 11 & 4 \\ 19 & 3 & 0 & 5 & 5 \\ 11 & 7 & 0 & 5 & 1 \end{pmatrix}$$

Step 2. Subtract the smallest element of each column from every element of the corresponding column, we get the following reduced matrix

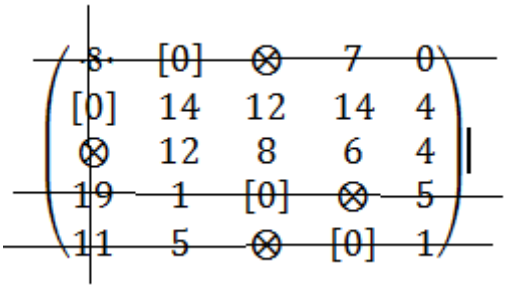
$$\begin{pmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{pmatrix}$$

Step 3. Starting with row one, we make assignment in a single zero and cross out all other zeros in the column marked, we get

$$\begin{pmatrix} 8 & [0] & \otimes & 7 & 0 \\ [0] & 14 & 12 & 14 & 4 \\ \otimes & 12 & 8 & 6 & 4 \\ 19 & 1 & [0] & \otimes & 5 \\ 11 & 5 & \otimes & [0] & 1 \end{pmatrix}$$

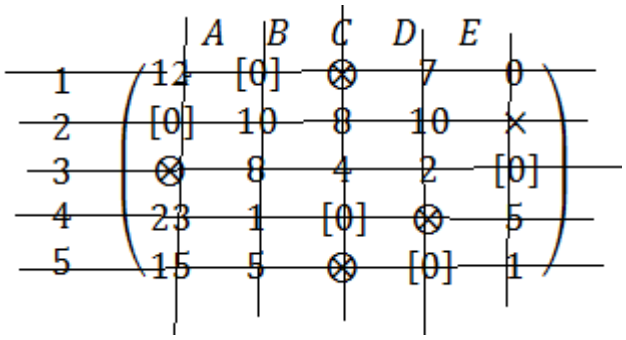
Here row three and column 5 do not have any assignment.

Step 4. Draw the minimum number of horizontal and vertical lines wchich cover all the zeroes as follows



Since the number of lines (4) is less than the order of matrix (5), the solution is not optimal.

Step 5. The least uncovered element 4 is subtracted from all the uncovered elements and added to the intersection of the elements, we get the following reduced matrix.



Optimum assignment is

$$1 \to B, \qquad 2 \to A, \qquad 3 \to E, \qquad 4 \to C, \qquad 5 \to D$$

i.e.,

$38 + 40 + 37 + 41 + 35 = 191$ (in hundred rupees).

Maximum sales would be Rs 19,100.