Answer on Question #60707 – Math – Calculus

Question

Evaluate

$$\int_0^2 dr \int_0^{\left(\frac{\pi}{2}\right)} d\theta \frac{\sin 2\theta}{\sqrt{(\cos^2\theta + 16)}}$$

Solution

$$\sin 2\theta = 2\sin\theta\cos\theta,$$

$\sin 2\theta\, d\theta = 2\sin\theta\cos\theta\, d\theta = -2\cos\theta\, d\cos\theta = -d(\cos^2\theta + 16),$

$\int_0^2 dr = (r)_0^2 = 2 - 0 = 2.$

$$I = \int_0^2 dr \int_0^{\left(\frac{\pi}{2}\right)} d\theta \frac{\sin 2\theta}{\sqrt{(\cos^2\theta + 16)}} =$$

$$= -2\int_0^{\left(\frac{\pi}{2}\right)} \frac{d(\cos^2\theta + 16)}{\sqrt{(\cos^2\theta + 16)}} = -2\int_0^{\left(\frac{\pi}{2}\right)} (\cos^2\theta + 16)^{-\frac{1}{2}} d(\cos^2\theta + 16) = -2\left(\frac{\sqrt{(\cos^2\theta + 16)}}{\frac{1}{2}}\right)_{\left(\frac{\pi}{2}\right)}^{0} =$$

$= 4\left(\sqrt{1+16} - \sqrt{0+16}\right) = 16\left(\sqrt{1+\frac{1}{16}} - 1\right) = 16\left(\frac{\sqrt{17}}{4} - 1\right) = 4\sqrt{17} - 16.$

Answer: $4\sqrt{17} - 16.$