Answer on Question #60704 – Math – Calculus

Question

1. Evaluate $\int_{\frac{1}{2}}^{1}\int_{0}^{2x}\cos(\pi x^2)dydx$.

Solution

We shall integrate with respect to y at first:

$$\int_{\frac{1}{2}}^{1}\int_{0}^{2x}\cos(\pi x^2)dydx=\int_{\frac{1}{2}}^{1}\cos(\pi x^2)(y)|_{y=0}^{y=2x}dx=\int_{\frac{1}{2}}^{1}\cos(\pi x^2)(2x-0)dx=$$

$$=\int_{\frac{1}{2}}^{1}2\mathrm{x}\cos(\pi x^2)dx.$$

Now we can integrate with respect to x:

$$\int_{\frac{1}{2}}^{1}2\mathrm{x}\cos(\pi x^2)dx=|2xdx=d(x^2)|=\int_{\frac{1}{2}}^{1}\cos(\pi x^2)d(x^2)=\int_{\frac{1}{2}}^{1}\frac{1}{\pi}\cos(\pi x^2)d(\pi x^2)=$$

$$=\frac{1}{\pi}\sin(\pi x^2)\,|_{\frac{1}{2}}^{1}=\frac{1}{\pi}\left(\sin(\pi)-\sin\left(\frac{\pi}{4}\right)\right)=\frac{1}{\pi}(0-\frac{1}{\sqrt{2}})=-\frac{1}{\pi\sqrt{2}}.$$

Answer:

$$\int_{\frac{1}{2}}^{1}\int_{0}^{2x}\cos(\pi x^2)dydx=-\frac{1}{\pi\sqrt{2}}.$$