Answer on Question #60656 - Math - Statistics and Probability

Question

a) The distribution of marks obtained by 500 candidates in a particular exam is given below:

Marks more than: 0 10 20 30 40 50
Number of candidates 500 460 400 200 100 30

Calculate the lower quartile marks. If 70% of the candidates pass in the exam, find the minimum marks obtained by a pass candidate. (5)

Solution

a)

The lower quartile marks:

$$\text{the lower half of the data} \begin{cases} 0 \\ \mathbf{10} - \text{The lower quartile value} \\ 20 \end{cases}$$

Median

$$\text{the upper half of the data} \begin{cases} 30 \\ 40 \\ 50 \end{cases}$$

$Q_1 = 10$

If 70% of the candidates pass in the exam, find
the minimum marks obtained by a pass candidate:

$70\%\text{ of the candidates } = 500 \cdot 70\% = 350\text{ candidates}$

400 (80%) people passed the exam with the assessment of 20 or above. That's why the the minimum marks of 350 candidates is 20.

The minimum marks obtained by a pass candidate: 20.

Answer:

The lower quartile marks: $Q_1 = 10$
The minimum marks obtained by a pass candidate: 20.

b) An analysis of monthly wages paid to the workers of two firms A and B belonging to the same industry gives the following results:

Firm A Firm B
Number of workers 500 600
Average daily wages ` 186 ` 175
Variance of distribution of wages 81 100

i) Which firm, A or B, has a large wage bill?
ii) In which firm, A or B, is there greater variability in individual wages?
iii) Find the average daily wage and the variance of the distribution of wages of all the workers in the firms A and B taken together.

i) Which firm, A or B, has a large wage bill?

Number of workers on Firm A: $N_{F_a} = 500$
Average daily wages on Firm A: $A_{F_a} = 186$
Wage bill on Firm A: $N_{F_a} \cdot A_{F_a} = 500 \cdot 186 = 93000$

Number of workers on Firm B: $N_{F_b} = 600$
Average daily wages on Firm B: $A_{F_b} = 175$
Wage bill on Firm B: $N_{F_b} \cdot A_{F_b} = 600 \cdot 175 = 105000$

$$93000 < 105000$$

So, firm B has larger wage bill.

ii) In which firm, A or B, is there greater variability in individual wages?

<u>For firm A</u>
Variance of distribution of wages on Firm A: $V_{F_a} = 81$
Standard deviation: $\sigma = \sqrt{V_{F_a}} = 9$
Coefficient of variation $= \frac{\sigma}{A_{F_a}} \cdot 100 = \frac{9}{186} \cdot 100 = 4.8387$
<u>For firm B</u>
Variance of distribution of wages on Firm B: $V_{F_b} = 100$
Standard deviation: $\sigma = \sqrt{V_{F_b}} = 10$
Coefficient of variation $= \frac{\sigma}{A_{F_b}} \cdot 100 = \frac{10}{175} \cdot 100 = 5.7143$

Variability of the firm depends upon coefficient of variation.
Higher the coefficient of variation, higher the variability.
Coefficient of variation of firm B is higher .
Hence , firm B shows greater variability in individual wages.

iii) Find the average daily wage and the variance of the distribution of wages of all the workers in the firms A and B taken together.

Average daily wage $\bar{x} = \frac{N_{F_a} \cdot A_{F_a} + N_{F_b} \cdot A_{F_b}}{N_{F_a} + N_{F_b}} = \frac{93000+105000}{500+600} = 180$

Variance of the distribution of wages:
The standard deviations of two series containing n_1, and n_2, members are σ_1, and σ_2, respectively, being measure for their respective means $\overline{x_1}$, and $\overline{x_2}$. If the two series are grouped together as one series of $(n_1 + n_2)$ members, show that the standard deviation σ of this series, measured from its mean $\bar{x}$, is given by:

$$\sigma^2 = \frac{n_1\left({\sigma_1}^2 + {d_1}^2\right) + n_2\left({\sigma_2}^2 + {d_2}^2\right)}{(n_1 + n_2)}$$

where $d_1 = A_{F_a} - \bar{x}$ and $d_2 = A_{F_b} - \bar{x}$
This formula we obtain the following simplification of expression the mean square deviation of two series taken together:

$$\sigma^2 = \frac{1}{n_1 + n_2} \sum_{1}^{n_1+n_2} f(x-a)^2$$

$$d_1 = 186 - 180 = 6 \quad \Rightarrow \quad {d_1}^2 = 36$$
$$d_2 = 175 - 180 = -5 \ \Rightarrow \quad {d_2}^2 = 25$$
$$\sigma^2 = \frac{500(81 + 36) + 600(100 + 25)}{(500 + 600)} = \frac{58500 + 75000}{1100} = 121.36$$

Answer:

i) firm B has larger wage bill.

ii) firm B shows greater variability in individual wages.

iii) Average daily wage $\bar{x} = 180$, variance of the distribution of wages $\sigma^2 = 121.36$