Answer on Question #60650 – Math – Statistics and Probability

Question

a) Let X be normally distributed and the mean of X is 12 and S.D. is 4. Find

i) $P[X \geq 20]$

ii) $P[X \leq 20]$

iii) $p[0 \leq X \leq 12]$

iv) x' when $p[X > x'] = 0.24$

v) x_0' and x_1', when $P(x_0' < X < x_1') = 0.50$ and $P(X > x_1') = 0.25$.

Solution

i)

$$P[X \geq 20] = P\left[Z \geq \frac{20-12}{4}\right] = P[Z \geq 2] = P[Z \leq -2] = 0.0228$$

ii)

$$P[X \leq 20] = 1 - P[X \geq 20] = 1 - 0.0228 = 0.9772$$

iii)

$$p[0 \leq X \leq 12] = P\left[\frac{0-12}{4} \leq Z \leq \frac{12-12}{4}\right] = P[-3 \leq Z \leq 0] = P[Z \leq 0] - P[Z \leq -3]$$
$$= 0.5 - 0.0013 = 0.4987$$

iv) $p[X > x'] = 0.24$

$$p[Z > z'] = 0.24 \rightarrow p[Z < -z'] = 0.24$$

$$-z' = -0.71 \rightarrow z' = 0.71$$

$$x' = 12 + 0.71 \cdot 4 = 14.8$$

v)

$$p[z_0 < Z < z_1] = 1 - p[Z > z_1] - p[Z < z_0] = 1 - 0.25 - p[Z < z_0] = 0.5$$

$$p[Z < z_0] = 0.25 \rightarrow z_0 = -0.67$$

$$p[Z < z_0] = p[Z > z_1] \rightarrow z_1 = -z_0 = 0.67$$

$$x_0 = 12 - 0.67 \cdot 4 = 9.3$$

$$x_1 = 12 + 0.67 \cdot 4 = 14.7$$

Question

b) A die is thrown 9000 times and the outcome of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one and find the limits between which the probability of a throw of 3 or 4 lies.

Solution

H_0: the die is unbiased, i.e. $P = \frac{1}{3}$ (the probability of a throw of 3 or 4).

$H_a: P \neq \frac{1}{3}$.

Two-tailed test is to be used.

Let LOS be 5%. Therefore, $z_\alpha = 1.96$.

Although we may test the significance of the difference between the sample and population proportions, we shall test the significance of the difference between the number of successes X in the sample and that in the population.

When n is large, X follows $N(np, \sqrt{nPQ})$.

$$z = \frac{X - nP}{\sqrt{nPQ}} = \frac{3240 - 9000 \cdot \frac{1}{3}}{\sqrt{9000 \cdot \frac{1}{3} \cdot \frac{2}{3}}} = 5.37$$

$$|z| > z_\alpha$$

Therefore, the difference between X and nP is significant, i.e., H_0 is rejected.

That is, the dice cannot be regarded as unbiased.

If X follows $N(\mu, \sigma)$, then $P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = 0.9974$.

The limits $\mu \pm 3\sigma$ are considered as the extreme (confidence) limits within which X lies.

Accordingly, the extreme limits for P are given by

$$\frac{|P - p|}{\sqrt{\frac{pq}{n}}} \leq 3$$

$$p - 3\sqrt{\frac{pq}{n}} \leq P \leq p + 3\sqrt{\frac{pq}{n}}$$

$$0.36 - 3\sqrt{\frac{0.36 \cdot 0.64}{9000}} \leq P \leq 0.36 + 3\sqrt{\frac{0.36 \cdot 0.64}{9000}}$$

$$0.345 \leq P \leq 0.375.$$