Answer on Question #60649 – Math – Statistics and Probability

Question

a) Let the random variable X have the following distribution:

P(X = 0)= P(X =2) = p

P(X =1) =1− 2 p, where 0 ≤ p ≤1.

for what value of p is the var(X) a maximum? Justify.

Solution

$$Var(X) = M(X^2) - \left(M(X)\right)^2 = \sum_{i=1}^{3} x_i^2 p_i - \left(\sum_{i=1}^{3} x_i p_i\right)^2 =$$

$$= (0^2 \cdot p + 1^2 \cdot (1 - 2p) + 2^2 \cdot p) - (0 \cdot p + 1 \cdot (1 - 2p) + 2 \cdot p)^2$$

$$= 1 - 2p + 4p - 1 = 2p$$

$$\max Var(X) = \max\left\{2p, 0 \le p \le \frac{1}{2}\right\} = 1$$

$VarX$ is maximum for $p = \frac{1}{2}$.

Question

b) In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter ' p ' of the distribution.

Solution

For a binomial distribution, the probability of X successes in n trials is given by

$$\binom{n}{x} p^x q^{n-x}.$$

Probability of one success in 5 independent trials is

$$\binom{5}{1} pq^4 = 0.4096.$$

Probability of two successes in 5 independent trials is

$$\binom{5}{2} p^2 q^2 = 0.2048.$$

Now find:

$$\frac{\binom{5}{1} pq^4}{\binom{5}{2} p^2 q^2} = \frac{0.4096}{0.2048}$$

$$\frac{5q}{10p} = 2$$

On the other hand,

$$p + q = 1$$

Then

$$4p = q = 1 - p \to \quad 4p + p = 1 \quad \to \quad 5p = 1 \to \ p = \frac{1}{5}.$$

Answer: 0.200.

Question

c) Fit a Poisson distribution to the following data:

x_i	0	1	2	3	4
p_i	$\frac{109}{200}$	$\frac{65}{200}$	$\frac{22}{200}$	$\frac{3}{200}$	$\frac{1}{200}$

Solution

Calculating the mean and variance:

$$E(N) = \big(M(X)\big) = \sum_{i=1}^{5} x_i p_i = \frac{1}{200}(0 \cdot 109 + 1 \cdot 65 + 2 \cdot 22 + 3 \cdot 3 + 4 \cdot 1 = 0.61$$

$$Var(X) = M(X^2) - \big(M(X)\big)^2 = \sum_{i=1}^{5} x_i^2 p_i - \left(\sum_{i=1}^{5} x_i p_i\right)^2$$
$$= \frac{1}{200}(0^2 \cdot 109 + 1^2 \cdot 65 + 2^2 \cdot 22 + 3^2 \cdot 3 + 4^2 \cdot 1) - 0.61^2 = 0.98 - 0.37$$
$$= 0.61$$

Since $E\ (N)$ = $Var\ (X)$, this can be modeled as a Poisson distribution because in this distribution

E(N)=$Var\ (X)$ = λ, and here we can assign λ = 0.61:

$$\Pr(\ N = k\ ) = \frac{\lambda^k}{k!} e^{-\lambda} = \frac{0.61^k}{k!} e^{-0.61}, for\ k \geq 0.$$