

## Answer on Question #60648 – Math – Statistics and Probability

### Question

a) For a distribution, the mean is 10, variance is 16, coefficient of skewness is +1 and coefficient of kurtosis is 4. Obtain the first four moments about the origin i.e., zero.

Comment upon the nature of distribution.

### Solution

Mean =  $m'_1 = 10$ , variance =  $m_2 = 16$ , coefficient of skewness =  $\gamma_1 = +1$ , coefficient of kurtosis is  $\beta_2 = 4$ .

First moment about the origin is

$$\text{Mean} = m'_1 = 10.$$

Next,

$$\text{Variance} = m_2 = 16 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2.$$

Second moment about the origin is

$$m'_2 = \frac{\sum x^2}{n} = m_2 + \mu_1^2 = \text{Variance} + \left(\frac{\sum x}{n}\right)^2 = 16 + 10^2 = \mathbf{116}$$

Coefficient of skewness is +1:

$$\gamma_1 = +1 = \frac{m_3}{m_2^{\frac{3}{2}}} \rightarrow m_3 = m_2^{\frac{3}{2}} = 16^{\frac{3}{2}} = 64$$

Coefficient of kurtosis is 4:

$$\beta_2 = 4$$

$$\frac{m_4}{m_2^2} = 4 \rightarrow m_4 = 4(16)^2 = 1024$$

Third moment about the origin is

$$m'_3 = m_3 + 3m_2m'_1 + m_1'^3 = 64 + 3(16)(10) + 10^3 = \mathbf{1544}$$

Fourth moment about the origin is

$$m'_4 = m_4 + 4m_3m'_1 + 6m_2m_1'^2 + m_1'^4 = 1024 + 4(64)(10) + 6(16)10^2 + 10^4 = \mathbf{23184}$$

It is a positively skewed distribution.

### Question

b) Calculate the correlation coefficient for the following heights (in inches) of father (X) and their sons (Y) :

X : 65 66 67 67 68 69 70 72

Y : 67 68 65 68 72 72 69 71

### Solution

The correlation coefficient is given by

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

Here

$$n = 8$$

$$\sum xy = 65 \cdot 67 + 66 \cdot 68 + 67 \cdot 65 + 67 \cdot 68 + 68 \cdot 72 + 69 \cdot 72 + 70 \cdot 69 + 72 \cdot 71 = 37560$$

$$\sum y = 67 + 68 + 65 + 68 + 72 + 72 + 69 + 71 = 552$$

$$\sum x = 65 + 66 + 67 + 67 + 68 + 69 + 70 + 72 = 544$$

$$\sum y^2 = 67^2 + 68^2 + 65^2 + 68^2 + 72^2 + 72^2 + 69^2 + 71^2 = 38132$$

$$\sum x^2 = 65^2 + 66^2 + 67^2 + 67^2 + 68^2 + 69^2 + 70^2 + 72^2 = 37028$$

The correlation coefficient is

$$r = \frac{(8)(37560) - (544)(552)}{\sqrt{8(38132) - (552)^2} \sqrt{8(37028) - (544)^2}} = +0.603.$$