Answer on Question #60647—Math – Statistics and Probability

Which of the following statements are true or false? Justify your answer.

Question

i) For normal distribution, mean deviation about mean is greater than quartile deviation.

Solution

i) True.

For normal distribution, mean deviation about mean $\frac{4}{5}\sigma$ is greater than quartile deviation $\frac{2}{3}\sigma$.

$$\frac{4}{5} > \frac{2}{3}.$$

Question

ii) In uniform distribution, the percentile points are equal-spaced.

Solution

ii) True.

The cumulative distribution function is

$$P(x, a, b) = \frac{x - a}{b - a}$$

is linear. Thus, if $P(x_i, a, b) - P(x_j, a, b) = const$ then $x_i - x_j = const$.

Question

iii) A high positive value of coefficient of correlation between the increase in cigarette smoking and increase in lung cancer establishes that cigarette smoking is responsible for lung cancer.

Solution

iii) True.

A high positive value of coefficient of correlation between the increase in cigarette smoking and increase in lung cancer means that that there is a strong positive linear relationship between cigarette smoking and lung cancer.

Question

iv) In the case of Poisson distribution with parameter λ, $\bar{x}$ is sufficient for λ .

Solution

iv) True.

We can write the joint probability mass function as

$$f(x_1, x_2, \dots, x_n; \lambda) = (e^{-n\lambda}\lambda^{n\bar{x}}) \cdot \left(\frac{1}{x_1!\, x_2! \dots x_n!}\right)$$

Therefore, the Factorization Theorem tells us that $Y = \bar{x}$ is a sufficient statistic for λ.

Question

v) The sum of two independent Poisson variables is also a Poisson variable.

Solution

v) True.

$P(X+Y=k) = \sum_{i=0}^{k} P(X+Y=k, X=i) = \sum_{i=0}^{k} P(Y=k-i, X=i) = \sum_{i=0}^{k} P(Y=k-i)\, P(X=i) =$
$= \sum_{i=0}^{k} e^{-\mu} \frac{\mu^{k-i}}{(k-i)!} e^{-\lambda} \frac{\lambda^{i}}{(i)!} = \frac{e^{-(\mu+\lambda)}}{k!} \sum_{i=0}^{k} \frac{k!}{i!(k-i)!} \mu^{k-i} \lambda^{i} = \frac{e^{-(\mu+\lambda)}}{k!} \sum_{i=0}^{k} \binom{k}{i} \mu^{k-i} \lambda^{i} == \frac{e^{-(\mu+\lambda)}}{k!} (\mu+\lambda)^{k}.$

Hence the sum of two independent Poisson variables is also a Poisson variable.

Question

vi) If Var(X) = Var(Y) and 2X + Y and X − Y are independent, then X and Y are independent.

Solution

vi) False.

If 2X + Y and X − Y are independent, then

$$Var\big((2X+Y)+(X-Y)\big) = Var\big((2X+Y)\big) + Var((X-Y))$$

$$Var(3X) = 9Var(X) = 4Var(X) + Var(Y) + 4Cov(X,Y) + Var(X) + Var(Y) - 2Cov(X,Y)$$

$$9Var(X) = 5Var(X) + 2Var(Y) + 2Cov(X,Y)$$

$$Var(X) = Var(Y) \rightarrow 2Cov(X,Y) = 2Var(X) \rightarrow Cov(X,Y) \neq 0$$

Thus, X and Y are not independent.

Question

vii) Mutually exclusive events are not independent.

Solution

vii) True.

If A and B are independent events, then

$$P(A\ and\ B) = P(A)\,P(B)$$

that is, the probability that both A and B occur is equal to the probability that A occurs times the probability that B occurs.

If A and B are mutually exclusive events, then

$$P(A\ and\ B) = 0$$

that is, the probability that both A and B occur is zero. Clearly, if A and B are nontrivial events ($P(A)\ and\ P(B)$ are nonzero), then they cannot be both independent and mutually exclusive.

Question

viii) For any two events A and B, $P(A\cap B)$ cannot be less than either P(A) or P(B) .

Solution

viii) False.

Consider a basic example:

What is the probability of drawing a red card from a normal 52 card deck and rolling an even number on a normal 6-side die?

Since drawing a red card and rolling an even number are entirely unrelated and have no bearing on each other, the two events are independent and you can use the simplified formula for the intersection of events.

Let event A = drawing a red card

Let event B = rolling an even number

$$P(A) = 1/2$$
$$P(B) = 1/2$$
$$P(A\cap B) = P(A)P(B)$$
$$P(A\cap B) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}.$$
$$P(A\cap B) < P(A)\ and\ P(A\cap B) < P(B)$$

Question

ix) If P(A) =0, then A = ϕ.

Solution

ix) True.

$$P(A) = \frac{n(A)}{n(S)} = 0 \rightarrow n(A) = 0 \rightarrow A = \phi.$$

Question

x) If $P(A \cap B) = \frac{1}{2}$, $P(\bar{A} \cap \bar{B}) = \frac{1}{2}$ and $2\mathrm{P(A)} = \mathrm{P(B)} = \mathrm{p}$, then the value of p is 1/4 .

Solution

x) False.

$$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$P(A \cup B) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(A \cup B) = P(\mathrm{A}) + P(\mathrm{B}) - P(A \cap B)$$

$$\frac{1}{2} = \frac{p}{2} + p - \frac{1}{2}$$

$$p = \frac{2}{3}$$

Answer: i) True; ii) True; iii) True; iv) True; v) True; vi) False; vii) True; viii) False; ix) True; x) False.