Answer on Question #60646 – Math – Statistics and Probability

Question

a) If 6n tickets numbered 1 ,1,0 ,2 K, 6n − are placed in a bag and three are drawn out, show that the probability that the sum of the numbers on them is equal to 6n is 3n /(6n – 1)(6 n – 2) .

Solution

First, note that 3 tickets can be drawn out from 6n tickets in $\binom{6n}{3}$ different ways. Hence,

$$n(S) = \binom{6n}{3} = \frac{6n(6n-1)(6n-2)}{3\cdot 2\cdot 1} = n(6n-1)(6n-2)$$

Next, let E denote the event that the sum of the numbers on the three randomly chosen tickets is 6n.

If 0 is the smallest of the three selected numbers, then there are 3n-1 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 0 & 1 & 6n-1 \\ \vdots & \ddots & \vdots \\ 0 & 3n-1 & 3n+1 \end{bmatrix}$$

If 1 is the smallest of the three selected numbers, then there are 3n-2 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 1 & 2 & 6n-3 \\ \vdots & \ddots & \vdots \\ 1 & 3n-1 & 3n \end{bmatrix}$$

If 2 is the smallest of the three selected numbers, then there are 3n-4 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 2 & 3 & 6n-5 \\ \vdots & \ddots & \vdots \\ 2 & 3n-2 & 3n \end{bmatrix}$$

If 3 is the smallest of the three selected numbers, then there are 3n-5 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 3 & 4 & 6n-7 \\ \vdots & \ddots & \vdots \\ 3 & 3n-2 & 3n-1 \end{bmatrix}$$

This process can be continued successively.

If 2n-4 is the smallest of the three selected numbers, then there are 5 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 2n-4 & 2n-3 & 2n+7 \\ 2n-4 & 2n-2 & 2n+6 \\ 2n-4 & 2n-1 & 2n+5 \\ 2n-4 & 2n & 2n+4 \\ 2n-4 & 2n+1 & 2n+3 \end{bmatrix}$$

If 2n-3 is the smallest of the three selected numbers, then there are 4 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 2n-3 & 2n-2 & 2n+5 \\ 2n-3 & 2n-1 & 2n+4 \\ 2n-3 & 2n & 2n+3 \\ 2n-3 & 2n+1 & 2n+2 \end{bmatrix}$$

If 2n-2 is the smallest of the three selected numbers, then there are 2 possible sets of triplets each having total equal to 6n.

$$\begin{bmatrix} 2n-2 & 2n-1 & 2n+3 \\ 2n-2 & 2n & 2n+2 \end{bmatrix}$$

If 2n-1 is the smallest of the three selected numbers, then there is 1 possible set of triplets each having total equal to 6n.

$$\begin{bmatrix} 2n-2 & 2n-1 & 2n+3 \\ 2n-2 & 2n & 2n+2 \end{bmatrix}$$

$$\begin{bmatrix} 2n-1 & 2n & 2n+1 \end{bmatrix}$$

Since the cases listed above are mutually exclusive, it follows that

$$\begin{aligned} n(E) &= 1+2+4+5+\cdots+(3n-5)+(3n-4)+(3n-2)+(3n-1) \\ &= [1+4+\cdots+(3n-5)+(3n-2)]+[2+5+\cdots+(3n-4)+(3n-1)] \\ &= \frac{n}{2}[2+3(n-1)]+\frac{n}{2}[4+3(n-1)] = \frac{n}{2}[6+6(n-1)] = 3n^2 \end{aligned}$$

Thus,

$$P(E) = \frac{n(E)}{n(S)} = \frac{3n^2}{n(6n-1)(6n-2)} = \frac{3n}{(6n-1)(6n-2)}$$

Question

b) From a bag containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of four balls transferred 3 are white and 1 is black?

Solution

$$P(3W1B) = \frac{\binom{3}{3}\binom{5}{1}}{\binom{3+5}{4}} = \frac{\binom{3}{3}\binom{5}{1}}{\binom{8}{4}} = \frac{\frac{3}{3!\,(3-3)!}\cdot\frac{5!}{1!\,(5-1)!}}{\frac{8!}{4!\,(8-4)!}} = \frac{1\cdot 5}{\frac{8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4}} = \frac{1}{14}.$$