Answer on Question #60618 – Math – Statistics and Probability

Question

Grade 11 students enrolled in schools in the LSB jurisdiction have had an average score of 63% on the exam with a standard deviation of 11%. Using the data from a clustered random sample of students from amongst the school board (see table below), determine if this year's performance (2012) on the math exam was significantly lower than the historical average. Conduct a complete hypothesis test using a confidence level of 95%.

A complete hypothesis test includes:

a) The null and alternative hypotheses, Ho and Ha.

b) The critical value (confidence coefficient).

c) The test statistic (including sample mean and standard deviation.

Student Score

A 72

B 85

C 41

D 50

E 89

F 75

G 62

H 64

I 52

J 60

K 39

L 48

M 70

N 61

O 40

P 79

Solution

a) The null hypothesis is $H_0\colon \mu \geq 63$, the alternative hypothesis is $H_a\colon \mu < 63$.

b) The critical value is

$$t_{crit} = t(\alpha, df) = t(1 - CL, n - 1) = t(1 - 0.95, 16 - 1) = t(0.05, 15)$$

From t-table:

$$t(0.05, 15) = 1.753.$$

c)

$$\sum x_i = 987,$$

$$\sum x_i^2 = 64667.$$

The sample mean is

$$\bar{x} = \frac{\sum x_i}{n} = \frac{987}{16} = 61.6875.$$

The sample standard deviation is

$$s = \sqrt{\frac{\sum x_i^2 - n(\bar{x})^2}{n-1}} = \sqrt{\frac{64667 - 16(61.6875)^2}{16-1}} = 15.8775.$$

The test statistic is

$$T = \frac{\bar{x} - 63}{\frac{s}{\sqrt{n}}} = \frac{61.6875 - 63}{\frac{15.8775}{\sqrt{16}}} = -0.331.$$

The test statistic is greater than $-t_{crit}$. So, we don't reject the null hypothesis. There is no sufficient evidence at 5% significance level to conclude that this year's performance (2012) on the math exam was significantly lower than the historical average.