

Answer on Question #60616 – Math– Calculus

Question

differentiate following functions:

1. $y = 2x^2 e^{2x}$

2. $y = (x^2 - 2) / (x^2 + 2)$

3. $y = 2\sin(3x^2 + 2x)$

Solution

$$(c)' = 0; (x)' = 1;$$

$$(\sin(x))' = \cos(x);$$

Apply the table formulae of derivatives:

$$(x^n)' = nx^{n-1};$$

$$(e^x)' = e^x$$

1. This problem requires using the product rule for the derivatives:

$$\text{if } f(x) = g(x)h(x) \text{ then } f'(x) = g'(x)h(x) + g(x)h'(x),$$

and the chain rule:

$$\text{if } f(x) = g(h(x)) \text{ then } f'(x) = g'(h(x))h'(x)$$

$$y = 2x^2 e^{2x};$$

$$y' = (2x^2 e^{2x})' = (2x^2)' \cdot e^{2x} + 2x^2 \cdot (e^{2x})' = 2 \cdot 2x \cdot e^{2x} + 2x^2 \cdot e^{2x} \cdot (2x)' = 4x e^{2x} + 2x^2 \cdot e^{2x} \cdot 2 = 4x e^{2x} + 4x^2 e^{2x} = 4x e^{2x} (1 + x)$$

Answer: $4x e^{2x} (1 + x)$.

2. This problem requires using the quotient rule for the derivatives:

$$\text{if } f(x) = \frac{g(x)}{h(x)} \text{ then } f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h^2(x)}$$

$$y = \frac{(x^2 - 2)}{(x^2 + 2)}$$

$$y' = \left(\frac{(x^2 - 2)}{(x^2 + 2)} \right)' = \frac{(x^2 - 2)'(x^2 + 2) - (x^2 - 2)(x^2 + 2)'}{(x^2 + 2)^2} = \frac{2x(x^2 + 2) - 2x(x^2 - 2)}{(x^2 + 2)^2} = \frac{2x(x^2 + 2 - x^2 + 2)}{(x^2 + 2)^2} = \frac{2x \cdot 4}{(x^2 + 2)^2} = \frac{8x}{(x^2 + 2)^2}$$

Answer: $\frac{8x}{(x^2 + 2)^2}$.

3. This problem requires using the chain rule for the derivatives:

$$\text{if } f(x) = g(h(x)) \text{ then } f'(x) = g'(h(x))h'(x)$$

$$y = 2\sin(3x^2 + 2x);$$

$$y' = (2\sin(3x^2 + 2x))' = 2\sin'(3x^2 + 2x) \cdot (3x^2 + 2x)' = 2\cos(3x^2 + 2x) \cdot (3 \cdot 2x + 2) = 2 \cdot 2 \cdot (3x + 1) \cdot \cos(3x^2 + 2x) = 4(3x + 1) \cdot \cos(3x^2 + 2x)$$

Answer: $4(3x + 1) \cdot \cos(3x^2 + 2x)$.