

Answer on Question #60615 – Math – Statistics and Probability

Question

- Determine the direction of the hypothesis test (one-sided left, one-sided right, bidirectional)
- Determine the test statistic (z^* or t^*) and the p-value for each of the following situations and
- Determine if they would cause the rejection of the null hypothesis if the confidence level was set at 95% in each case. (Hint: be wary of the sample size) [2 points each]:

a) $H_0: \mu = 50$ mL, $H_a: \mu \neq 50$ mL, sample mean = 48.1 mL, sample standard deviation = 5, $n = 40$;

b) $H_0: \mu \leq 8.4$ mL, $H_a: \mu > 8.4$ mL, sample mean = 10 mL, $s = 3.5$ mL, $n = 25$;

c) $H_0: \mu \geq 20^\circ\text{C}$, $H_a: \mu < 20^\circ\text{C}$, sample mean = 17.1 $^\circ\text{C}$, $s = 4.6^\circ\text{C}$, $n = 12$;

d) $H_0: \mu = 357$ s, $H_a: \mu \neq 380$ s, sample mean = 410 s, $s = 75$, $n = 40$;

e) $H_0: \mu \leq 46$ units, $H_a: \mu > 46$ units, sample mean = 50 units, $s = 9.5$, $n = 41$.

Solution

The general rule for when to use a t^* statistic is when our sample size meets the following two requirements:

- The sample size is below 30
- The population standard deviation is unknown (estimated from your sample data)

In all our cases we know only sample standard deviation and therefore should use t^* statistic.

a) Bidirectional; Test statistic t^* :

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{48.1 - 50}{\frac{5}{\sqrt{40}}} = -2.40.$$

$$p = 0.021 < 0.05. \text{ Reject the null hypothesis.}$$

b) One –sided right; Test statistic t^* :

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{10 - 8.4}{\frac{3.5}{\sqrt{25}}} = 2.29.$$

$$p = 0.016 < 0.05. \text{ Reject the null hypothesis.}$$

c) One –sided left; Test statistic t*:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{17.1 - 20}{\frac{4.6}{\sqrt{12}}} = -2.18.$$

$p = 0.026 < 0.05$. Reject the null hypothesis.

d) Bidirectional; Test statistic t*:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{410 - 380}{\frac{75}{\sqrt{40}}} = 2.53.$$

$p = 0.016 < 0.05$. Reject the null hypothesis.

e) One –sided right; Test statistic t*:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{50 - 46}{\frac{9.5}{\sqrt{41}}} = 2.70.$$

$p = 0.005 < 0.05$. Reject the null hypothesis.