Question

Evaluate cot 45° without using a calculator by using ratios in a reference triangle.

Solution
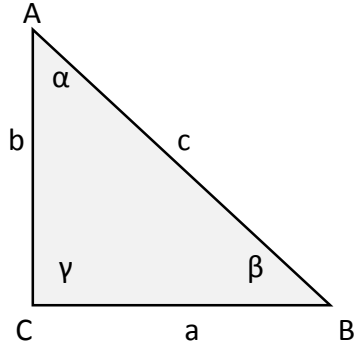
Given

$\alpha = 45°$,

$\beta = 45°$,

$\gamma = 90°$,

triangle ΔABC is isosceles, because two angles α, β of the triangle being equal.

Then $a = b$, because triangle ΔABC is isosceles. Triangle ΔABC is right, because $\gamma = 90°$.

By Pythagorean theorem,

$$c = \sqrt{a^2 + b^2} = \sqrt{a^2 + a^2} = a\sqrt{1 + 1} = a\sqrt{2}.$$

Using definitions of trigonometric functions

$\sin \alpha = \dfrac{a}{c} = \dfrac{a}{a\sqrt{2}} = \dfrac{1}{\sqrt{2}}$,

$\cos \alpha = \dfrac{b}{c} = \dfrac{a}{a\sqrt{2}} = \dfrac{1}{\sqrt{2}}$,

$\cot \alpha = \dfrac{\sin \alpha}{\cos \alpha} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1.$

Given $\alpha = 45°$, formula $\cot 45° = 1$ has been proved.

Answer: $\cot 45° = 1.$