Question

The Department of Foreign Languages of a liberal arts college conducted a survey of its recent graduates to determine the foreign language courses they had taken while undergraduates at the college. Of the 530 graduates

209 had at least one year of Spanish.

176 had at least one year of French.

142 had at least one year of German.

47 had at least one year of Spanish and French.

31 had at least one year of Spanish and German.

27 had at least one year of French and German.

7 had at least one year of all three languages.

(a) How many of the graduates had at least 1 yr of at least one of the three languages?

(b) How many of the graduates had at least 1 yr of exactly one of the three languages?

(c) How many of the graduates had less than 1 yr of any of the three languages?

Solution

Denote $S = $ "$Spanish$", $F = $ "$French$", $G = $ "$German$", $U = $ "$All\ graduates$".

Given $|S| = 209, |F| = 176, |G| = 142,\ |S \cap F| = 47, |S \cap G| = 31,\ |F \cap G| = 27,$

$|S \cap G \cap F| = 7,$

calculate

$|S \cap G \cap \bar{F}| = |S \cap G| - |S \cap G \cap F| = 31 - 7 = 24,$

$|S \cap \bar{G} \cap F| = |S \cap F| - |S \cap G \cap F| = 47 - 7 = 40,$

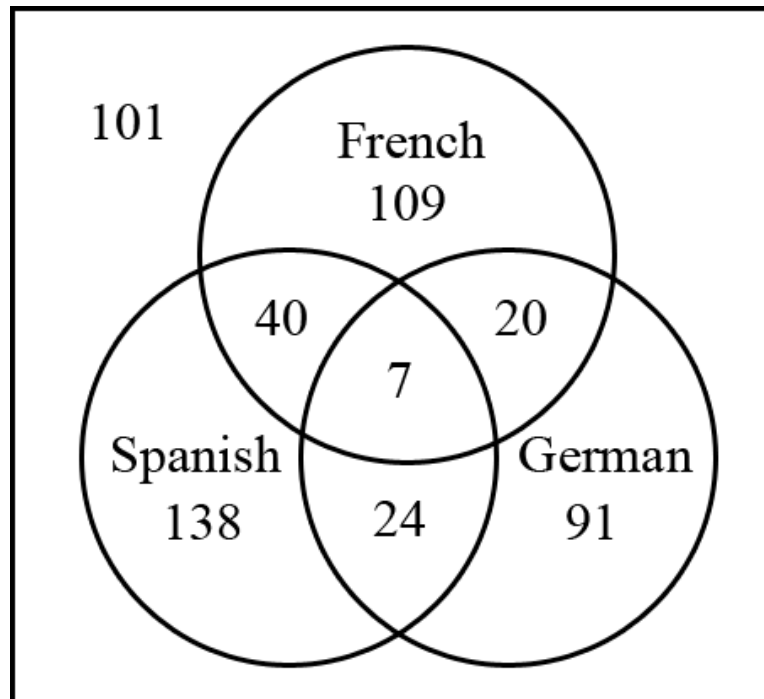$|\bar{S} \cap G \cap F| = |F \cap G| - |S \cap G \cap F| = 27 - 7 = 20,$

$|S \cap \bar{F} \cap \bar{G}| = |S| - |S \cap \bar{G} \cap F| - |S \cap G \cap \bar{F}| - |S \cap G \cap F| = 209 - 40 - 24 - 7 = 138,$

$|\bar{S} \cap F \cap \bar{G}| = |F| - |S \cap \bar{G} \cap F| - |\bar{S} \cap G \cap F| - |S \cap G \cap F| = 176 - 40 - 20 - 7 =$

$= 109,$

$|\bar{S} \cap \bar{F} \cap G| = |G| - |S \cap G \cap \bar{F}| - |\bar{S} \cap G \cap F| - |S \cap G \cap F| = 142 - 24 - 20 - 7 = 91,$

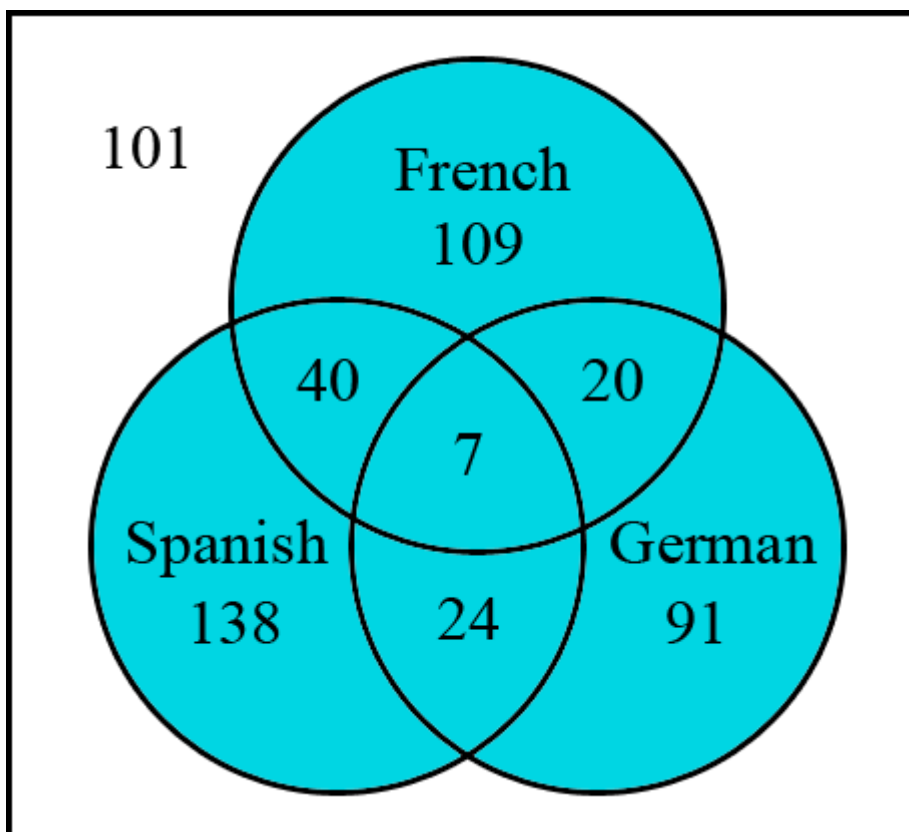$$|\bar{S} \cap \bar{F} \cap \bar{G}| = |U| - |S \cap \bar{F} \cap \bar{G}| - |\bar{S} \cap F \cap \bar{G}| - |\bar{S} \cap \bar{F} \cap G| - |S \cap \bar{G} \cap F| -$$

$$-|\bar{S} \cap G \cap F| - |S \cap G \cap \bar{F}| - |S \cap G \cap F| = 530 - 138 - 109 - 91 - 40 - 20 - 24 -$$
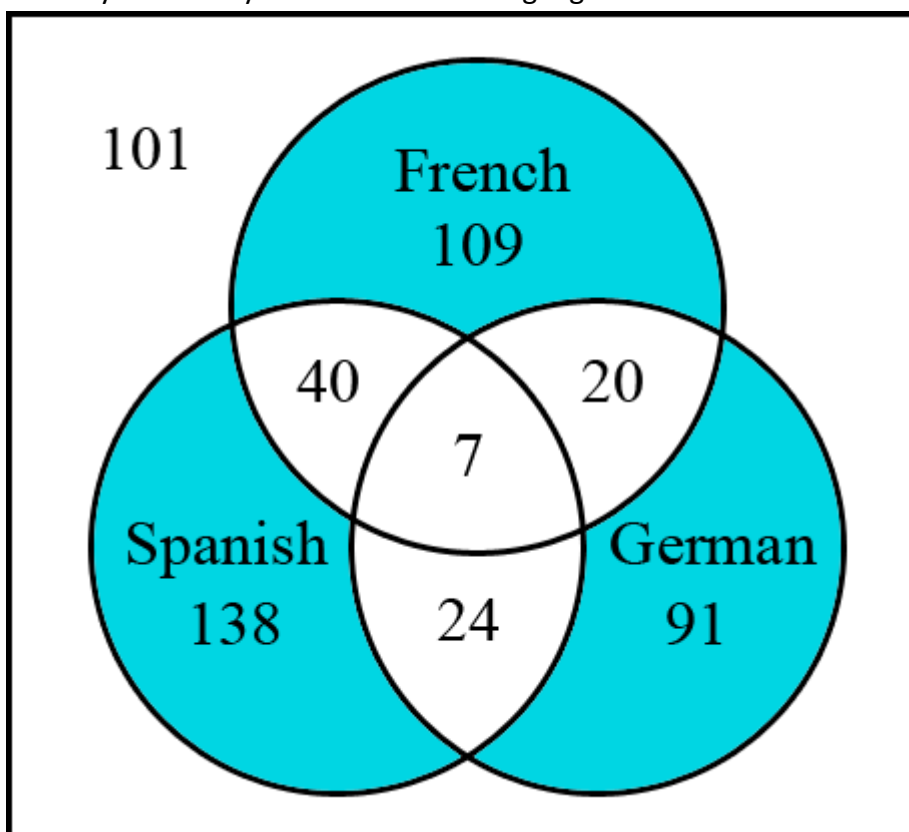
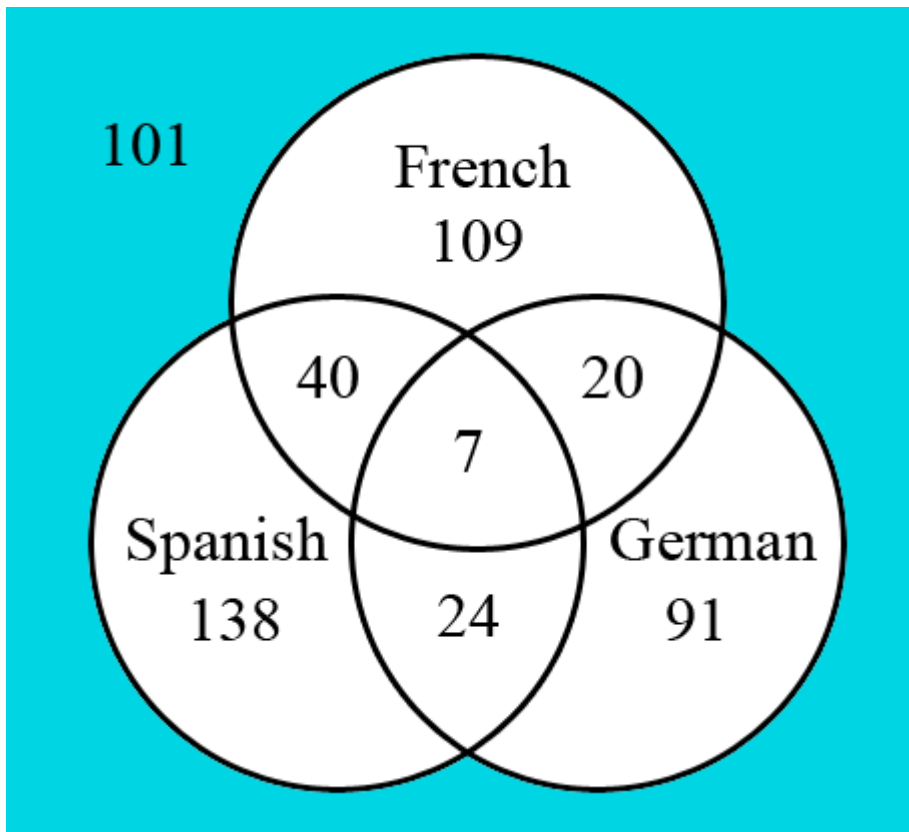$$-7 = 101$$

Venn diagram for the given case:

(a)

$|S \cup F \cup G| = |U| - |\bar{S} \cap \bar{F} \cap \bar{G}| = 530 - 101 = 429$ graduates had at least 1 yr of at least one of the three languages

(b) $|\bar{S} \cap F \cap \bar{G}| + |S \cap \bar{F} \cap \bar{G}| + |\bar{S} \cap \bar{F} \cap G| = 109 + 138 + 91 = 338$ graduates had at least 1 yr of exactly one of the three languages

(c) $|\bar{S} \cap \bar{F} \cap \bar{G}| = 101$ graduates had less than 1 yr of any of the three languages

Answer: (a) 429; **(b)** 338; **(c)** 101.