

## Answer on Question #60547 – Math – Statistics and Probability

### Question

The organizer of the Montreal International Art Exhibit is trying to determine its optimal operating hours for its next one-day exhibition. Studies have shown that the arrival times at any given exhibition form a normal distribution with the average time that visitors arrive being 2 hours and 56 minutes after doors open, with a standard deviation of 48 minutes.

- a) If the organizer sets the opening of the exhibition at 10:00 a.m., at what time would they expect 95% of the visitors to have arrived?
- b) If the organizer sets the opening of the exhibition at 9:00 a.m., at what time after the doors open will only 15% of the visitors have arrived?
- c) At what time should the organizer open the exhibition if they would like 70% of the visitors to have arrived by 1:00 p.m. so that they can award the first door prize?

### Solution

2 h 56 min = 176 min

In the given case, one should consider normal distribution with  $\mu = 176$  and  $\sigma = 48$ .

- a) The fact “95% visitors arrived” means the cutoff score for the top 5% of arrival times, or  $P(x > X) = 0.05$ ;  $P(x < X) = 0.95$

The z-score associated with the given probability value can be obtained either from the standard normal table or by using the technology (NORM.INV()) function of MS Excel).

For  $p = 0.95$ ,  $z = 1.645$ .

Converting z-score to the arrival time value:

$$X = \mu + z\sigma = 176 + 1.645 \times 48 = 255 \text{ min} = 4 \text{ h } 15 \text{ min.}$$

If the exhibition opens at 10 am, 95% of visitors will arrive by 2:15 pm.

- b) The fact “95% visitors arrived” means  $P(x < X) = 0.15$ .

For  $p = 0.15$ ,  $z = -1.036$ .

$$X = \mu + z\sigma = 176 - 1.036 \times 48 = 126 \text{ min} = 2 \text{ h } 6 \text{ min.}$$

If the exhibition opens at 9 am, 95% of visitors will arrive by 11:06 am.

- c)  $P(x < X) = 0.7$ .

For  $p = 0.7$ ,  $z = 0.524$ .

$$X = \mu + z\sigma = 176 + 0.524 \times 48 = 201 \text{ min} = 3 \text{ h } 21 \text{ min.}$$

If the organizer needs 70% arrival rate by 1 pm, he needs to open the exhibition at 9:39 am.

**Answer: a) 2:15 pm; b) 11:06 am; c) 9:39 am.**