Answer on Question #60533 – Math – Trigonometry

Question

2cosa-sina=x,cosa-2sina=y, then prove that 2xsquare+y square-2xy=5

Solution

$$\begin{cases} 2\cos\alpha - \sin\alpha = x \\ \cos\alpha - 2\sin\alpha = y \end{cases}$$

$$x^2 = 4cos^2\alpha + sin^2\alpha - 4cos\alpha \cdot sin\alpha$$

$$y^2 = cos^2\alpha + 4sin^2\alpha - 4cos\alpha \cdot sin\alpha$$

$$xy = (2cos\alpha - sin\alpha)(cos\alpha - 2sin\alpha) = 2cos^2\alpha - 4cos\alpha \cdot sin\alpha - cos\alpha \cdot sin\alpha + 2sin^2\alpha$$
$$= 2 - 5cos\alpha \cdot sin\alpha$$

$$2x^2 + y^2 - 2xy = 2(4cos^2\alpha + sin^2\alpha - 4cos\alpha \cdot sin\alpha) +$$

$$+(cos^2\alpha + 4sin^2\alpha - 4cos\alpha \cdot sin\alpha) - 2(2 - 5cos\alpha \cdot sin\alpha) =$$
$$= 8cos^2\alpha + 2sin^2\alpha - 8cos\alpha \cdot sin\alpha + cos^2\alpha + 4sin^2\alpha - 4cos\alpha \cdot sin\alpha - 4 +$$

$$+10cos\alpha \cdot sin\alpha = 9cos^2\alpha + 6sin^2\alpha - 2cos\alpha \cdot sin\alpha = 3cos^2\alpha + 6(cos^2\alpha + sin^2\alpha) - 2cos\alpha \cdot sin\alpha =$$

$$= 3cos^2\alpha + 6 - 2cos\alpha \cdot sin\alpha = 5.$$

Hence

$$3cos^2\alpha + 1 - 2cos\alpha \cdot sin\alpha = 0,$$

$$3cos^2\alpha + cos^2\alpha + sin^2\alpha - 2cos\alpha \cdot sin\alpha = 0,$$

$$4cos^2\alpha - 2cos\alpha \cdot sin\alpha + sin^2\alpha = 0 \quad \textbf{(1)}$$

If $cos\alpha = 0$ then it follows from (1) that $sin\alpha = 0$, which does not hold, because $cos^2\alpha + sin^2\alpha = 1$ according to the Pythagorean identity.

If $cos\alpha \neq 0$ then dividing by $cos^2\alpha$ in (1) one gets

$$tan^2\alpha - 2tan\alpha + 4 = 0 \qquad \textbf{(2)}$$

Equation (2) does not have real solutions, because its discriminant is negative:

$$D = 2^2 - 4 \cdot 1 \cdot 4 < 0\ .$$

Thus, $2x^2 + y^2 - 2xy \neq 5$ when $x = 2cos\alpha - sin\alpha$, $y = cos\alpha - 2sin\alpha$.