## Answer on Question #60452 – Math – Real Analysis

### Question

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + n(n + 1)(n + 2) = n(n + 1)(n + 2)(n + 3)/4 \qquad (1)$$

### Solution

Consider the proof of (1) by mathematical induction.

1. **Basis of the Induction**.

Show that the statement (1) holds for $n = 1$:
the left-hand side is $1\cdot2\cdot3=6$,
the right-hand side is $1\cdot(1+1)\cdot(1+2)\cdot(1+3)/4=2\cdot3\cdot4/4=6$,
hence 6=6 is true.

2. **Induction hypothesis.**

Assume the statement (1) holds for some $n$=k, where k>1:

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + k(k + 1)(k + 2) = k(k + 1)(k + 2)(k + 3)/4. \qquad (2)$$

3. **Induction step.**

It must then be shown that the statement (1) holds for $n$=k+1, that is,

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + (k+1)(k+1+1)(k+1+2) = (k+1)(k+1+1)(k+1+2)(k+1+3)/4;$$

i. e.

$$1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + (k+1)(k+2)(k+3) = (k+1)(k+2)(k+3)(k+4)/4; \qquad (3)$$

Consider the left-hand side of (3):
$1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + (k+1)(k+2)(k+3) =$
= { include the last two terms}=
$=1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + k(k + 1)(k + 2) + (k+1)(k+2)(k+3)=$
= {using the induction hypothesis (2) for the first k terms}=

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3) =$$

={factor out the common multipliers (k+1), (k+2), (k+3)}=

$$= (k+1)(k+2)(k+3)\cdot\left(\frac{k}{4}+1\right) = (k+1)(k+2)(k+3)\cdot\left(\frac{k+4}{4}\right) = \frac{(k+1)(k+2)(k+3)(k+4)}{4},$$

so we deduced the right-hand side of (3).

Thus, from the assumption of the validity of formula (1) for $n$=k it follows that it is also valid for $n$=k+1.

4. According to the principle of mathematical induction, formula (1) is proved for all natural numbers.