

Answer on Question #60349 – Math – Statistics and Probability

Question

TubeView, a digital television distribution company based in Gatineau (QC), hires a marketing firm to gauge the satisfaction levels of their customers. The CEO of TubeView has claimed that past polls have suggested that the customer satisfaction level is at 90%.

If this claim was true, what is the probability that in a random sample of 12 customers:

- a) Exactly 8 customers are satisfied with TubeView's service?
- b) At least 8 customers are satisfied with TubeView's service?
- c) All of the customers they contact are satisfied with TubeView's service?
- d) At most 2 customers are not satisfied with TubeView's service?
- e) $P(5 \leq X < 9)$ where X = satisfied customer?

Solution

Verifying conditions required to apply the normal approximation for the binomial distribution:

$$np = 12 \times 0.9 = 10.8 > 5 \text{ passed}$$

$$nq = 12 \times 0.1 = 1.2 < 5 \text{ failed}$$

Therefore, one cannot use the normal approximation for the binomial distribution.

The probabilities can be calculated using the binomial formula:

$$P(x) = {}_n C_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

$$\text{a) } P(x=8) = \frac{12!}{(12-8)!8!} \times 0.9^8 \times 0.1^{12-8} = 0.0213$$

$$\text{b) } P(x \geq 8) = P(x=8) + P(x=9) + P(x=10) + P(x=11) + P(x=12)$$

$$P(x=9) = \frac{12!}{(12-9)!9!} \times 0.9^9 \times 0.1^{12-9} = 0.0852$$

$$P(x=10) = \frac{12!}{(12-10)!10!} \times 0.9^{10} \times 0.1^{12-10} = 0.2301$$

$$P(x=11) = \frac{12!}{(12-11)!11!} \times 0.9^{11} \times 0.1^{12-11} = 0.3766$$

$$P(x=12) = \frac{12!}{(12-12)!12!} \times 0.9^{12} \times 0.1^{12-12} = 0.2824$$

$$P(x \geq 8) = 0.0213 + 0.0852 + 0.2301 + 0.3766 + 0.2824 = 0.9956$$

c)
$$P(x=12) = \frac{12!}{(12-12)!12!} \times 0.9^{12} \times 0.1^{12-12} = 0.2824$$

d) The event "at most 2 customers are not satisfied" is equal to "at least 10 customers satisfied"

$$P(x \geq 10) = P(x=10) + P(x=11) + P(x=12)$$

$$P(x \geq 10) = 0.2301 + 0.3766 + 0.2824 = 0.8891$$

e)
$$P(5 \leq x < 9) = P(x=5) + P(x=6) + P(x=7) + P(x=8)$$

$$P(x=5) = \frac{12!}{(12-5)!5!} \times 0.9^5 \times 0.1^{12-5} = 0$$

$$P(x=6) = \frac{12!}{(12-6)!6!} \times 0.9^6 \times 0.1^{12-6} = 0.0005$$

$$P(x=7) = \frac{12!}{(12-7)!7!} \times 0.9^7 \times 0.1^{12-7} = 0.0038$$

$$P(x=8) = \frac{12!}{(12-8)!8!} \times 0.9^8 \times 0.1^{12-8} = 0.0213$$

$$P(5 \leq x < 9) = 0 + 0.0005 + 0.0038 + 0.0213 = 0.0256.$$

Answer:

a) 0.0213;

b) 0.9956;

c) 0.2824;

d) 0.8891;

e) 0.0256.