Answer on Question #60309 – Math – Calculus

Question

Give an example of a situation in which composite differentiation might be used. Give examples of functions that might be applicable in your situation, and show how the relevant rates of change might be calculated.

Solution

Here is an example of a situation in which composite differentiation might be used. Consider a spherical balloon being inflated so that its radius is increasing at a rate of 3 centimeters per seconds. Let *R* denote the radius of the balloon in centimeters, *t* denote time in seconds, and *V* denote the volume of the balloon in cubic centimeters, then we know that

$$V = \frac{4}{3}\pi R^3,$$

$$R = 3t.$$

Moreover, we can see that, as a function of *t*,

$$V = \frac{4}{3}\pi(3t)^3.$$

We use composite differentiation if we have composite function or functions of a function.

To differentiate composite functions we have to use the chain rule:

$$\big(f\big(g(x)\big)\big)' = (f \circ g)' = f'\big(g(x)\big)g'(x)$$

In our example,

$$\frac{dV}{dt} = \frac{dV}{dR} \cdot \frac{dR}{dt},$$

$$\frac{dV}{dR} = 4\pi R^2,$$

$$\frac{dR}{dt} = 3.$$

Then

$$\frac{dV}{dt} = 4\pi R^2 \cdot 3 = 12\pi(3t)^2 = 108\pi t^2.$$