Answer on Question #60045 – Math – Abstract Algebra

Question

Prove that
(i) H={a+ib ЄC, a2+b2=1} is a subgroup of C, where C is complex number.
(ii) H be a set of real number a+b√2 where a,b ϵ Q. Show that H be a subgroup of non-zero real no. under *.

Proof

(i) Let's prove that H is a subgroup under multiplication.
If r1 = a1 + ib1, r2 = a2 + ib2 ϵ H, then
r1*r2 = (a1 + ib1)*(a2 + ib2) = a1*a2 + ib1*a2 + ia1*b2 - b1*b2 =
=a1*a2 - b1*b2 + i (b1*a2 + a1*b2) , (a1*a2 - b1*b2)^2 + (b1*a2 + a1*b2)^2 =
= (a1^2 + b1^ n2)(a2^2 + b2^2) = 1 and
r1*r2 ϵ H .
Since (a1 - i b1)(a1 + ib1) = a1^2 + b1^2 = 1, then r1^(-1) = a1 - ib1 and r1 ϵ H.
From these relations we obtain that H is a subgroup.

(ii) If r1 = a1 + b1sqrt(2), r2 = a2 + b2sqrt(2), then
r1*r2 = a1*a2 + 2b1*b2 + (a1*b2 + b1*a2)sqrt(2) ϵ H.
Since r1^(-1) = -a1/(2b1^2 - a1^2) + b1/(2b1^2 - a1^2)sqrt(2) ϵ H
(r1^(-1)*r1 = 1/(2b1^2 - a1^2) (-a1 + b1sqrt(2))(a1 + b1sqrt(2)) = 1/(2b1^2 - a1^2) (2b1^2 - a1^2) = 1), then H is a subgroup under *.