Answer on Question #59972 – Math – Statistics and Probability

Question

For every n>1 the random variable Xn is exponential with parameter λn, where λn→λ>0 and X is an exponential with parameter λ, then show that Xn converges in distribution to X

Solution

We say that $\{X_n\}$ converges in distribution to the random variable X if

$$\lim_{n\to\infty} F_n(t) = F(t),$$

at every value t where F is continuous cumulative distribution function.

$$F_n(t) = 1 - e^{-\lambda_n t}, for\ t \geq 0$$

$$F(t) = 1 - e^{-\lambda t}, for\ t \geq 0$$

$$\lim_{n\to\infty} F_n(t) = \lim_{n\to\infty}\left(1 - e^{-\lambda_n t}\right) = 1 - \lim_{n\to\infty}\left(e^{-\lambda_n t}\right) = 1 - \exp\left(-t \lim_{n\to\infty} \lambda_n\right) = 1 - \exp(-t\lambda) = F(t),$$

for $t \geq 0$.

Thus, $\{X_n\}$ converges in distribution to the random variable X.