

**Answer on Question #59960 – Math – Statistics and Probability**

**Question**

Construct a 95% confidence interval for the population mean of hourly wage. What sample size would you need if you wanted the confidence interval to have a width of at most \$1 on either side of the sample mean (so \$2 in total)?

Population mean: 5.185577655

Hourly wage:

1.878125

2.0125

2.058333

2.510417

2.752404

2.754464

3.007813

3.127404

3.188101

3.360902

3.367789

3.800481

3.822917

3.848558

3.851923

4.165865

4.357778

4.810096

5.071875

5.164063

5.5025

5.771635

5.771635

5.8675  
 5.985043  
 7.503125  
 8.079327  
 12.5025  
 12.65825  
 15.62891

**Solution**

A 95% confidence interval for the population mean of hourly wage is

$$CI = \left( \bar{x} - t^* \frac{s}{\sqrt{n}}; \bar{x} + t^* \frac{s}{\sqrt{n}} \right),$$

$t^* = 2.045$  for 95% confidence level and  $n - 1 = 30 - 1 = 29$  df (degrees of freedom).

See <http://www.z-table.com/t-value-table.html>:

<b>cum. prob</b>	<b>t<sub>.50</sub></b>	<b>t<sub>.75</sub></b>	<b>t<sub>.80</sub></b>	<b>t<sub>.85</sub></b>	<b>t<sub>.90</sub></b>	<b>t<sub>.95</sub></b>	<b>t<sub>.975</sub></b>	<b>t<sub>.99</sub></b>	<b>t<sub>.995</sub></b>	<b>t<sub>.999</sub></b>	<b>t<sub>.9995</sub></b>
<b>one-tail</b>	<b>0.50</b>	<b>0.25</b>	<b>0.20</b>	<b>0.15</b>	<b>0.10</b>	<b>0.05</b>	<b>0.025</b>	<b>0.01</b>	<b>0.005</b>	<b>0.001</b>	<b>0.0005</b>
<b>two-tails</b>	<b>1.00</b>	<b>0.50</b>	<b>0.40</b>	<b>0.30</b>	<b>0.20</b>	<b>0.10</b>	<b>0.05</b>	<b>0.02</b>	<b>0.01</b>	<b>0.002</b>	<b>0.001</b>
<b>df</b>											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
<b>Z</b>	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	<b>Confidence Level</b>										

The population mean is

$$\bar{x} = 5.139407767.$$

The sample standard deviation is

$$s = 3.283247478.$$

A 95% confidence interval for the population mean of hourly wage is

$$\begin{aligned} CI &= \left( 5.139407767 - 2.045 \frac{3.283247478}{\sqrt{30}}; 5.139407767 + 2.045 \frac{3.283247478}{\sqrt{30}} \right) \\ &= (3.913423010; 6.365392523) \end{aligned}$$

The sample size would be

$$n = \left( \frac{z^* s}{E} \right)^2 = \left( \frac{1.960 \cdot 3.283247478}{1} \right)^2 = 41.4 \text{ round up to } 42.$$

**Answer:** (3.913423010; 6.365392523); 42.