Question

6. Suppose a fair coin is tossed three times, what is the probability that at least one head occurs?

Solution

Suppose a fair coin is tossed three times. We list all eight possible outcomes of this event: $\{HHH, HHT, HTH, THH, TTH, THT, HTT, TTT\}$. We see that there are 7 favorable outcomes of the event "at least one head occurs". Thus, the probability that at least one head occurs is equal to $\frac{7}{8} = 0.875$.

Answer: $\frac{7}{8} = 0.875$.

Question

7. Suppose a box contains 4 red balls and 3 black balls. Compute the probability that the second ball drawn is red if the first ball drawn was black.

Solution

After drawing the first black ball the box contains 4 red balls and 2 black balls. The probability that the second ball drawn is red if the first ball drawn was black will be equal to $\frac{4}{4+2} = \frac{2}{3}$.

Answer: $\frac{2}{3}$.

Question

8. There are three children in a family. If there is at least one boy and at most two boys in this family, what is the conditional probability that there are exactly two boys in this family?

Solution

We list all possible outcomes of the event "at least one boy and at most two boys": $\{1\ boy\ and\ 2\ girls; 2\ boys\ and\ 1\ girl\}$. The total number of outcomes is 2.

We see that there is only one favorable outcome $\{2\ boys\ and\ 1\ girl\}$ of the event "exactly two boys". Thus, the probability that there are exactly two boys in this family is equal to $\frac{1}{2} = 0.5$.

Answer: 0.5.

Question

9. Two women A, and B share an office with a single telephone. The probability that any call will be for A is $\frac{2}{3}$. Suppose that A is out of her office during the office hours half of the time and B one third. Find the probability that for any call during the working hours a call can be answered by the person being called.

Solution

Let $H_1 = \{a\ call\ will\ be\ for\ A\ \}$, and $H_2 = \{a\ call\ will\ be\ for\ B\}$.

It is given that $P(H_1) = \frac{2}{3}, P(H_2) = \frac{1}{3}$.

Let $C = \{a\ call\ will\ be\ answered\ by\ the\ person\ being\ called\}$.

It is given that $P(C/H_1) = \frac{1}{2}, P(C/H_2) = \frac{2}{3}$.

Then using the formula of total probability we find the required probability:

$$P(C) = P(C/H_1)P(H_1) + P(C/H_2)P(H_2) = \frac{1}{2} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} = \frac{5}{9}.$$

Answer: $\frac{5}{9}$.

Question

10. A box contains 6 balls numbered 1 to 6. A ball was drawn from the box at random. Find the probability that the number on the ball drawn was either 1, 2 or 6.

Solution

The number of the favorable outcomes of the event "the number on the ball drawn was either 1, 2 or 6" is 3.

The number of the total outcomes of the event "a ball was drawn from the box" is 6.

So the probability that the number on the ball drawn was either 1,2, or 6 is $\frac{3}{6} = \frac{1}{2} = 0.5$.

Answer: 0.5.