Answer on Question #59918 – Math – Statistics and Probability

Question

1 In how many ways can a family of 9 divide itself into 3 groups so that each group contains 3 persons?

Solution

We are seeking for unordered partitions $r_1 = 3, r_2 = 3\ r_3 = 3$.

The number of unordered partitions is

$$\frac{9!}{3!\,3!\,3!} \cdot \frac{1}{3!} = \frac{9!}{6^4} = 280$$

(since the three parts contain the same number of objects).

The same relationship that exists between permutation and combination exist between ordered and unordered partition of a set.

Answer: 280.

Question

2 Four digits numbers are to be formed using any of the digits 1, 2, 3, 4, 5, 6. If no repetition of digit is allowed how many 4-digit even numbers can be formed?

Solution

This is permutation of 4 digits from 6 digits. Therefore, the number of different permutations is

$$^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{(2)!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$$

Alternatively, we can reason as follows:

The first digit to be selected can be any of the six given digits, so $n_1 = 6$. The second digits to be select can be any of the remaining 5 digits (since no reparation is allowed) so $n_2 = 5$. Similarly, $n_3 = 4$ and $n_4 = 3$. Thus, the answer is

$$n_1 \cdot n_2 \cdot n_3 \cdot n_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360.$$

Answer: 360.

Question

3 A club consist of 10 men and 5 women, in how many ways can a committee of 6 consisting of 4 men and 2 women be chosen?

Solution

The 4 men can be chosen from the 10 men in $^{10}C_4$ ways, the 2 women can be chosen from the 5 women in $^5C_2$ ways. Hence the committee can be chosen in (by the fundamental principle of counting)

$$^{10}C_4\ ^5C_2 = \frac{10!}{6!\,4!} \cdot \frac{5!}{2!\,3!} = 2100\ ways$$

Answer: 2100 ways.

Question

4 Let A and B be any two events defined on the same sample space. Suppose P(A) = 0.3 and P(A B) = 0.6. Find P(B) such that A and B are independent.

Solution

For independent events we obtain

$$\mathrm{P(A\cap\ B)} = P(A)P(B)$$

If $\mathrm{P(A\cap\ B)} = 0.6$, then $P(B) = \frac{P(A\cap B)}{P(A)} = \frac{0.6}{0.3} = 2$, which is impossible, because $P(B) \leq 1$ has to be true.

If $\mathrm{P(A\cup\ B)} = 0.6$, then by the rule of addition,

$$\mathrm{P(A\ U\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\cap\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - P(A)P(B).$$

$$P(B) = \frac{\mathrm{P(A\ U\ B)} - \mathrm{P(A)}}{1 - P(A)} = \frac{0.6 - 0.3}{1 - 0.3} = \frac{3}{7}.$$

Answer: $\frac{3}{7}$.

Question

5 Let A and B be any two events defined on the same sample space. Suppose P(A) = 0.3 and P(A B) = 0.6. Find P(B) such that A and B are mutually exclusive.

Solution

By the rule of addition,

$$\mathrm{P(A\ U\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\cap\ B)}.$$

If A and B are mutually exclusive, then $\mathrm{P(A\cap\ B)} = 0$. It follows that

$$\mathrm{P(B)} = \mathrm{P(A\ U\ B)} - \mathrm{P(A)} = 0.6 - 0.3 = 0.3.$$

Answer: 0.3.