Answer on Question #59915 – Math – Statistics and Probability

Question

8 Let X have a uniform distribution on the interval [A,B]. Compute V(X)

Solution

Since x is uniformly distributed in [A,B], the probability density function is

$$f(x) = \begin{cases} \frac{1}{B-A}, & A \le x \le B, \\ 0, otherwise. \end{cases}$$

The mean is

$$\mu = \int_{-\infty}^{+\infty} f(x)dx = \int_A^B \frac{xdx}{B-A} = \frac{1}{B-A}\left(\frac{x^2}{2}\right)_A^B = \frac{1}{2}\frac{1}{B-A}(B^2 - A^2) = \frac{A+B}{2}.$$

The variance is

$$V(X) = \int_{-\infty}^{+\infty}(x-\mu)^2 f(x)dx = \int_A^B \frac{(x-\mu)^2 dx}{B-A} = = \frac{1}{B-A}\int_A^B (x-\mu)^2 d(x-\mu) = \frac{1}{3}\frac{1}{B-A}\left[\left(B - \frac{A+B}{2}\right)^3 - \left(A - \frac{A+B}{2}\right)^3\right] = \frac{(B-A)^2}{12}.$$

Answer: $V(X) = \frac{(B-A)^2}{12}$.

Question

9 Let X have a standard gamma distribution with α=7. Compute P(X<4 or X>6)

Solution

Let the cumulative distribution function of the standard gamma distribution with shape parameter α be $F(x)$.

Then

$$P(X < 4) = F(4);\ P(X > 6) = 1 - F(6).$$

$\mathrm{P}(\mathrm{X} < 4 \text{ or } \mathrm{X} > 6) = P(X < 4) + P(X > 6) = F(4) + 1 - F(6) = 0.1107 + 1 - 0.3937 = 0.7170.$

We used Excel function GAMMA.DIST to calculate the values of the cumulative distribution function:

$F(6)$=GAMMA.DIST(6,7,1,TRUE) and $F(4)$=GAMMA.DIST(4,7,1,TRUE).

Answer: $\mathrm{P}(\mathrm{X} < 4 \text{ or } \mathrm{X} > 6) = 0.7170.$

Question

10 Let X = the time between two successive arrivals at the drive –up window of a bank. If X has a exponential distribution with h=1 (which is identical to a standard gamma distribution with a=1). Compute the standard deviation of the time between successive arrivals.

Solution

The probability density function (pdf) of an exponential distribution is

$$f(x) = \begin{cases} he^{-hx}, x \ge 0, \\ 0, otherwise. \end{cases}$$

where $\frac{1}{h}$ is the mean.

In our case $f(x) = \begin{cases} e^{-x}, x \ge 0, \\ 0, otherwise. \end{cases}$

The variance is

$$V(X) = \int_0^\infty (x-1)^2 e^{-x} dx = |y = x - 1| = \int_{-1}^\infty y^2 e^{-y-1} dy = \frac{1}{e}\int_{-1}^\infty y^2 e^{-y} dy$$

$$\int_{-1}^{\infty} y^2 e^{-y} dy = -\int_{-1}^{\infty} y^2 de^{-y} = (-y^2 e^{-y})_{-1}^{\infty} + 2\int_{-1}^{\infty} ye^{-y} dy$$

$$(-y^2 e^{-y})_{-1}^{\infty} = e$$

$$\int_{-1}^{\infty} ye^{-y} dy = (-ye^{-y})_{-1}^{\infty} + \int_{-1}^{\infty} e^{-y} dy$$

$$(-ye^{-y})_{-1}^{\infty} = -e$$

$$\int_{-1}^{\infty} e^{-y} dy = (-e^{-y})_{-1}^{\infty} = e$$

$$V(X) = \frac{1}{e}[e + 2(e - e)] = 1$$

The standard deviation is

$$\sigma = \sqrt{V(X)} = \sqrt{1} = 1.$$

Answer: $\sigma = 1.$