

Answer on Question #59914 – Math – Statistics and Probability

Question

3. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerator is running. Suppose that 7 of these refrigerators have defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order. Let X be the number among the first 6 examined that have a defective compressor. What is the probability that X exceeds its mean value by more than 1 standard deviation?

Solution

It is the hypergeometric probability distribution with

$$N = 12, n = 6, M = 7, N - M = 5.$$

Its mean is

$$\mu = n \frac{M}{N} = 6 \frac{7}{12} = 3.5.$$

Its standard deviation is

$$\sigma = \sqrt{n \frac{M}{N} \frac{N-M}{N} \frac{N-n}{N-1}} = \sqrt{6 \frac{7}{12} \frac{5}{12} \frac{12-6}{12-1}} = 0.89,$$

hence

$$\mu + \sigma = 3.5 + 0.89 = 4.39$$

The probability that X exceeds its mean value by more than 1 standard deviation is

$$P(X > \mu + \sigma) = 1 - P(X < \mu + \sigma) = 1 - F(\mu + \sigma).$$

We used Excel function HYPGEOM.DIST.

$$F(4.39) = \text{HYPGEOM.DIST}(4.39, 6, 7, 12, \text{TRUE}) = 0.8788;$$

$$P(X > \mu + \sigma) = 1 - 0.8788 = 0.1212.$$

Answer: 0.1212.

Question

6. Let X be a continuous rv with cdf

$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x}{4} \left[1 + \ln \left(\frac{4}{x} \right) \right], & 0 < x \leq 4 \\ 1, & x > 4 \end{cases}$$

What is the pdf of X ?

Solution

The pdf of X is given by

$$f(x) = \frac{dF}{dx} = \begin{cases} \frac{1}{4} \left[1 + \ln\left(\frac{4}{x}\right) \right] + \frac{x}{4} \left[\frac{x}{4} \left(-\frac{4}{x^2} \right) \right], & 0 < x \leq 4 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{4} \ln\left(\frac{4}{x}\right), & 0 < x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Answer: $f(x) = \begin{cases} \frac{1}{4} \ln\left(\frac{4}{x}\right), & 0 < x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$

Question

7. In a study of plants, five characteristics are to be examined. If there are six recognizable differences in each of four characteristics and eight, recognizable difference in the remaining characteristics. How many plants can be distinguished by these five characteristics?

Solution

By the fundamental counting principle, the number of plants can be distinguished by these five characteristics is

$$N = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 8 = 10368.$$

Answer: 10368.