Answer on Question #59913 – Math – Statistics and Probability

Question

2. Components of a certain type are shipped to a supplier in batches of ten. Suppose that 50% of all such batches contain no defective components. 30% contain one defective component, and 20% contain two defective components. Two components from a batch are randomly selected and tested. What are the probabilities associated with 0, 1, and 2 defective components being in the batch under the condition that neither tested component is defective.

Solution

Let B_0 be the event that the batch has 0 defectives, B_1 be the event the batch has 1 defective, and B_2 be the event the batch has 2 defectives. Let D_0 be the event that neither selected component is defective.

$$P(B_0) = 0.5, P(B_1) = 0.3, P(B_2) = 0.2$$

The event D_0 can happen in three different ways: (i) Our batch of 10 is perfect, and we get no defectives in our sample of two; (ii) Our batch of 10 has 1 defective, but our sample of two misses them; (iii) Our batch has 2 defective, but our sample misses them.

For (i), the probability is $(0.5)(1)$.

For (ii), the probability that our batch has 1 defective is 0.3. Given that it has 1 defective, the probability that our sample misses it is $\frac{\binom{9}{2}}{\binom{10}{2}}$, which is $\frac{8}{10}$. So the probability of (ii) is $(0.3)\left(\frac{8}{10}\right)$.

For (iii), the probability our batch has 2 defective is 0.2. Given that it has 2 defective, the probability that our sample misses both is $\frac{\binom{8}{2}}{\binom{10}{2}}$, which is $\frac{56}{90}$. So the probability of (iii) is $(0.2)\left(\frac{56}{90}\right)$. We have therefore found that

$$P(D_0) = (0.5)(1) + (0.3)\left(\frac{8}{10}\right) + (0.2)\left(\frac{56}{90}\right).$$

We use the general conditional probability formula:

$$P(B_0|D_0) = \frac{P(B_0 \cap D_0)}{P(D_0)} = \frac{(0.5)(1)}{(0.5)(1)+(0.3)\left(\frac{8}{10}\right)+(0.2)\left(\frac{56}{90}\right)} = 0.5784;$$

$$P(B_1|D_0) = \frac{P(B_1 \cap D_0)}{P(D_0)} = \frac{(0.3)\left(\frac{8}{10}\right)}{(0.5)(1)+(0.3)\left(\frac{8}{10}\right)+(0.2)\left(\frac{56}{90}\right)} = 0.2776;$$

$$P(B_2|D_0) = \frac{P(B_2 \cap D_0)}{P(D_0)} = \frac{(0.2)\left(\frac{56}{90}\right)}{(0.5)(1)+(0.3)\left(\frac{8}{10}\right)+(0.2)\left(\frac{56}{90}\right)} = 0.1440.$$

Answer: $P(B_0|D_0) = 0.5784$, $P(B_1|D_0) = 0.2776$, $P(B_2|D_0) = 0.1440$.

Question

4. The error involved in making a certain measurement is a continuous rv X with pdf

f(x)={0.09375(4−x 2)0 −2≤x≤2otherwise

$$f(x) = \begin{cases} 0.09375(4 - \mathrm{x}^2), -2 \le x \le 2 \\ 0, \qquad otherwise \end{cases}$$

Compute P(-1<x<1).

Solution

$$\mathrm{P}(-1 < \mathrm{x} < 1) = \int_{-1}^{1} 0.09375(4 - \mathrm{x}^2) = 0.09375\left(4x - \frac{x^3}{3}\right)_{-1}^{1} = 0.09375\left(8 - \frac{2}{3}\right) = 0.6875\ .$$

Answer: 0.6875.