Answer on Question #59910 – Math – Calculus

Question

If A=5t2+tj−t3k and B=sinti−costj. Evaluate ddt(A·B).

Solution

$$\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)=\frac{d\vec{A}}{dt}\cdot\vec{B}+\vec{A}\cdot\frac{d\vec{B}}{dt}=$$

$$=\left(\frac{d(5t^2)}{dt}\hat{\imath}+\frac{dt}{dt}\hat{\jmath}-\frac{d(t^3)}{dt}\hat{k}\right)\cdot(\text{sint}\hat{\imath}-\text{cost}\hat{\jmath})+\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\cdot\left(\frac{d(\sin t)}{dt}\hat{\imath}+\frac{d(-\cos t)}{dt}\hat{\jmath}\right)=$$

$$=\left(10t\hat{\imath}+\hat{\jmath}-3t^2\hat{k}\right)\cdot(\text{sint}\hat{\imath}-\text{cost}\hat{\jmath})+\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\cdot(\text{cost}\hat{\imath}+\text{sint}\hat{\jmath})$$

$$=10t\sin t-\cos t++(-3t^2)\cdot 0+5t^2\cos t+tsin\,t+(-t^3)\cdot 0=$$

$$=5t^2\cos t+t\sin t+10t\sin t-\cos t=$$

$=(5t^2-1)\cos t+11t\sin t.$

Answer: $(5t^2-1)\cos t+11t\sin t.$

Question

If A=5t2+tj−t3k and B=sinti−costj. Evaluate ddt(A×B).

Solution

$$\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)=\frac{d\vec{A}}{dt}\times\vec{B}+\vec{A}\times\frac{d\vec{B}}{dt}=$$

$$=\left(\frac{d(5t^2)}{dt}\hat{\imath}+\frac{d(t)}{dt}\hat{\jmath}-\frac{d(t^3)}{dt}\hat{k}\right)\times(\text{sint}\hat{\imath}-\text{cost}\hat{\jmath})+\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\times\left(\frac{d(\sin t)}{dt}\hat{\imath}+\frac{d(-\cos t)}{dt}\hat{\jmath}\right)==$$

$$\left(10t\hat{\imath}+\hat{\jmath}-3t^2\hat{k}\right)\times(\text{sint}\hat{\imath}-\text{cost}\hat{\jmath})+\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\times(\text{cost}\hat{\imath}+\text{sint}\hat{\jmath})=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\10t&1&-3t^2\\\sin t&-\cos t&0\end{vmatrix}+$$

$$+\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\5t^2&t&-t^3\\\cos t&\sin t&0\end{vmatrix}=\left(-3t^2\cos t\,\hat{\imath}-3t^2\sin t\,\hat{\jmath}-(10t\cos t+\sin t)\hat{k}\right)+\left(t^3\sin t\,\hat{\imath}-t^3\cos t\,\hat{\jmath}+\right.$$

$$\left.+(5t^2\sin t-tcos\,t)\hat{k}\right)=(t^3\sin t-3t^2\cos t)\hat{\imath}-(t^3\cos t+3t^2\sin t)\,\hat{\jmath}+(5t^2\sin t-11tcos\,t-\text{sint})\hat{k}.$$

Answer: $(t^3\sin t-3t^2\cos t)\hat{\imath}-(t^3\cos t+3t^2\sin t)\,\hat{\jmath}+(5t^2\sin t-11tcos\,t-\text{sint})\hat{k}.$

Question

If A=5t2+tj−t3k and B=sinti−costj. Evaluate ddt(A·A)

Solution

$$\frac{d}{dt}(\vec{A}\cdot\vec{A})=\frac{d\vec{A}}{dt}\cdot\vec{A}+\vec{A}\cdot\frac{d\vec{A}}{dt}=2\vec{A}\cdot\frac{d\vec{A}}{dt}=$$

$$=2\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\cdot\left(\frac{d(5t^2)}{dt}\hat{\imath}+\frac{d(t)}{dt}\hat{\jmath}-\frac{d(t^3)}{dt}\hat{k}\right)=2\left(5t^2\hat{\imath}+t\hat{\jmath}-t^3\hat{k}\right)\cdot\left(10t\hat{\imath}+\hat{\jmath}-3t^2\hat{k}\right)=$$

$$=2\left(5t^2\cdot 10t+t\cdot 1-t^3\cdot(-3t^2)\right)=2(50t^3+t+3t^5)=6t^5+100t^3+2t.$$

Answer: $6t^5+100t^3+2t.$

Question

If A=sinui+cosuj+uk, B=cosui−sinuj−3k and C=2i+3j−k, evaluate ddu(A×(B×C)) at u=0.

Solution

$$(\vec{B}\times\vec{C})=\left(\cos u\hat{\imath}-\sin u\hat{\jmath}-3\hat{k}\right)\times\left(2\hat{\imath}+3\hat{\jmath}-\hat{k}\right)=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \cos u & -\sin u & -3\\ 2 & 3 & -1\end{vmatrix}=(9+\sin u)\hat{\imath}+$$
$$+(-6+\cos u)\hat{\jmath}+(3\cos u+2\sin u)\hat{k};$$

$$\vec{A}\times(\vec{B}\times\vec{C})=\left(\sin u\hat{\imath}+\cos u\hat{\jmath}+u\hat{k}\right)\times\left((9+\sin u)\hat{\imath}+(-6+\cos u)\hat{\jmath}+(3\cos u+2\sin u)\hat{k}\right)=$$

$$=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ \sin u & \cos u & u\\ 9+\sin u & -6+\cos u & 3\cos u+2\sin u\end{vmatrix}=(6u-u\cos u+3\cos^2 u+2\cos u\sin u)\hat{\imath}+$$
$$+(9u+u\sin u-2\sin^2 u-3\cos u\sin u)\hat{\jmath}+(-9\cos u-6\sin u)\hat{k};$$

$$\frac{d}{du}\vec{A}\times(\vec{B}\times\vec{C})=(6-\cos u+u\sin u+2\cos 2u-3\sin 2u)\hat{\imath}+(9+\sin u+u\cos u-3\cos 2u-$$
$$2\sin 2u)\hat{\jmath}+(9\sin u-6\cos u)\hat{k};$$

$$\left(\frac{d}{du}\vec{A}\times(\vec{B}\times\vec{C})\right)_{u=0}=(6-1+0+2-0)\hat{\imath}+(9+0+0-3-0)\hat{\jmath}+(0-6)\hat{k}=7\hat{\imath}+6\hat{\jmath}-6\hat{k}.$$

Answer: $7\hat{\imath}+6\hat{\jmath}-6\hat{k}.$

Question

Let A=x2yzi−2xz3j−xz2 and B=4zi+yj+4x2k, find ∂2∂x∂y(A×B) at (1,0,-2).

Solution

$$(\vec{A}\times\vec{B})=\left(x^2yz\hat{\imath}-2xz^3\hat{\jmath}-xz^2\hat{k}\right)\times\left(4z\hat{\imath}+y\hat{\jmath}+4x^2\hat{k}\right)=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ x^2yz & -2xz^3 & -xz^2\\ 4z & y & 4x^2\end{vmatrix}=$$

$$=(xyz^2-8x^3z^3)\hat{\imath}+(-4x^4yz-4xz^3)\hat{\jmath}+(x^2y^2z+8xz^4)\hat{k};$$

$$\frac{\partial}{\partial y}(\vec{A}\times\vec{B})=(xz^2)\hat{\imath}+(-4x^4z)\hat{\jmath}+(2x^2yz)\hat{k};$$

$$\frac{\partial^2}{\partial x\partial y}(\vec{A}\times\vec{B})=z^2\hat{\imath}-16x^3z\hat{\jmath}+4xyz\hat{k};$$

$$\left(\frac{\partial^2}{\partial x \partial y}\left(\vec{A} \times \vec{B}\right)\right)_{(1,0,-2)} = 4\hat{\imath} + 32\hat{\jmath} + 0\hat{k} = 4\hat{\imath} + 32\hat{\jmath}.$$

Answer: $4\hat{\imath} + 32\hat{\jmath}$.

Question

Solve d2Adt2−4dAdt−5A=0

Solution

$$\frac{d^2A}{dt^2} - 4\frac{dA}{dt} - 5A = 0;$$

$$A = e^{\lambda t};$$

$$\lambda^2 e^{\lambda t} - 4\lambda e^{\lambda t} - 5e^{\lambda t} = 0.$$

Dividing by $e^{\lambda t} \neq 0$ obtain

$$\lambda^2 - 4\lambda - 5 = 0 \rightarrow \lambda = -1 \ or \ \lambda = 5.$$

$A = C_1 e^{-t} + C_2 e^{5t}$, where C_1, C_2 are arbitrary real constants.

Answer: $A = C_1 e^{-t} + C_2 e^{5t}$.