Answer on Question #59908 – Math – Calculus

Question

Given that A=sinti+costj+tk , evaluate |||d2Adt2|||

Solution

$$\vec{A} = sint\hat{\imath} + cost\hat{\jmath} + t\hat{k} \to \frac{d\vec{A}}{dt} = \frac{d(\sin t)}{dt}\hat{\imath} + \frac{d(\cos t)}{dt}\hat{\jmath} + \frac{d(t)}{dt}\hat{k} = cost\hat{\imath} - sint\hat{\jmath} + \hat{k} \to$$

$$\frac{d^2\vec{A}}{dt^2} = \frac{d(\cos t)}{dt}\hat{\imath} + \frac{d(-\sin t)}{dt}\hat{\jmath} + \frac{d(1)}{dt}\hat{k} = -sint\hat{\imath} - cost\hat{\jmath} \to$$

$$\left|\frac{d^2\vec{A}}{dt^2}\right| = \sqrt{(-sint)^2 + (-cost)^2} = \sqrt{1} = 1.$$

Answer: $\left|\frac{d^2\vec{A}}{dt^2}\right| = 1.$

Question

A particle moves along a curve whose parameter equations are x=e−t, y=2cos3t, z=2sin3t. Find the magnitude of the acceleration at t=0

Solution

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ be the position vector of any point P(x,y,z). Then

$$\vec{r} = \mathrm{e}^{-\mathrm{t}}\hat{\imath} + 2\mathrm{cos3t}\hat{\jmath} + 2\mathrm{sin3t}\hat{k}.$$

Now, velocity is given by

$$\frac{d\vec{r}}{dt} = \frac{d(e^{-t})}{dt}\hat{\imath} + \frac{d(2\cos(3t))}{dt}\hat{\jmath} + \frac{d(2\sin 3t)}{dt}\hat{k} = -\mathrm{e}^{-\mathrm{t}}\hat{\imath} - 6\mathrm{sin3t}\hat{\jmath} + 6\mathrm{cos3t}\hat{k}.$$

Acceleration is given by

$$\frac{d^2\vec{r}}{dt^2} = \frac{d(-e^{-t})}{dt}\hat{\imath} + \frac{d(-6\sin 3t)}{dt}\hat{\jmath} + \frac{d(6\cos 3t)}{dt}\hat{k} = \mathrm{e}^{-\mathrm{t}}\hat{\imath} - 18\mathrm{cos3t}\hat{\jmath} - 18\mathrm{sin3t}\hat{k}.$$

Acceleration at t=0 is given by

$$\frac{d^2\vec{r}}{dt^2} = \left.\frac{d^2\vec{r}}{dt^2}\right|_{t=0} = \mathrm{e}^{-0}\hat{\imath} - 18\cos(3\cdot 0)\,\hat{\jmath} - 18\sin(3\cdot 0)\,\hat{k} = \hat{\imath} - 18\hat{\jmath}.$$

The magnitude of the acceleration at t=0 is

$$\left|\frac{d^2\vec{r}}{dt^2}\right|_{t=0} = \sqrt{1 + (-18)^2} = \sqrt{325} = 5\sqrt{13}.$$

Answer: $\left|\frac{d^2\vec{r}}{dt^2}\right|_{t=0} = 5\sqrt{13}$.

Question

A particle moves along the curve x=2t2, y=t2−4t and z=3t−5, where t is the time. Find the components of the velocity at t=1 in the direction i−3j+2k

Solution

Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ be the position vector of any point P(x,y,z). Then

$$\vec{r} = 2\mathrm{t}^2\hat{\imath} + (\mathrm{t}^2 - 4\mathrm{t})\hat{\jmath} + (3\mathrm{t} - 5)\hat{k}.$$

Now, velocity is given by

$$\frac{d\vec{r}}{dt} = \frac{d(2t^2)}{dt}\hat{\imath} + \frac{d(t^2-4t)}{dt}\hat{\jmath} + \frac{d(3t-5)}{dt}\hat{k} = 4\mathrm{t}\hat{\imath} + (2\mathrm{t} - 4)\hat{\jmath} + 3\hat{k}.$$

Velocity at t=1 is given by

$$\left(\frac{d\vec{r}}{dt}\right)_{t=1} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}.$$

Required component of velocity $\frac{d\vec{r}}{dt}$ in the direction $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ is

$$\left(\frac{d\vec{r}}{dt}\right)_{t=1} \cdot \frac{\hat{\imath} - 3\hat{\jmath} + 2\hat{k}}{|\hat{\imath} - 3\hat{\jmath} + 2\hat{k}|} = \left(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right) \cdot \frac{\hat{\imath} - 3\hat{\jmath} + 2\hat{k}}{\sqrt{1+9+4}} = \frac{1}{\sqrt{14}}(4 \cdot 1 + (-2) \cdot (-3) + 3 \cdot 2) =$$

$$= \frac{1}{\sqrt{14}}(4 + 6 + 6) = \frac{14}{\sqrt{14}} = \sqrt{14}.$$

Answer: $\sqrt{14}$.