Answer on Question #59825 – Math – Linear Algebra

Question

Consider the square matrix A

{3,-1,0}
A= ,{-1,2-1},
{0,-1,3}
(i) Find the inverse of the matrix A.

Solution

To calculate the inverse matrix, we write the matrix A by adding the identity matrix on the right:

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array}\right)$$

To find the inverse matrix, we use elementary transformations over the rows of the matrix. We transform the left-hand side of the resulting matrix to the identity one.

The first row is divided by 3:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array}\right)$$

Add the first row to the second one, the result is placed in the second row:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{5}{3} & -1 & \frac{1}{3} & 1 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array}\right)$$

The second row is divided by $\frac{5}{3}$:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -0.6 & 0.2 & 0.6 & 0 \\ 0 & -1 & 3 & 0 & 0 & 1 \end{array}\right)$$

Add the third row to the second row, the result is placed in the third row:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -0.6 & 0.2 & 0.6 & 0 \\ 0 & 0 & 2.4 & 0.2 & 0.6 & 1 \end{array}\right)$$

The third row is divided by 2.4:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -0.6 & 0.2 & 0.6 & 0 \\ 0 & 0 & 1 & \frac{1}{12} & \frac{1}{4} & \frac{5}{12} \end{array}\right)$$

Add the third row, multiplied by 0.6, to the second row, the result is placed in the second row:

$$\left(\begin{array}{ccc|ccc} 1 & \frac{-1}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0.25 & 0.75 & 0.25 \\ 0 & 0 & 1 & 1/12 & 0.25 & 5/12 \end{array}\right)$$

Add the second row, multiplied by 1/3, to the first row, the result is placed in the first row:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/12 & 0.25 & 1/12 \\ 0 & 1 & 0 & 0.25 & 0.75 & 0.25 \\ 0 & 0 & 1 & 1/12 & 0.25 & 5/12 \end{array}\right).$$

Thus, $A^{-1} = \begin{pmatrix} \frac{5}{12} & 0.25 & \frac{1}{12} \\ 0.25 & 0.75 & 0.25 \\ \frac{1}{12} & 0.25 & \frac{5}{12} \end{pmatrix}$ is the inverse matrix.

Answer: $A^{-1} = \begin{pmatrix} \frac{5}{12} & 0.25 & \frac{1}{12} \\ 0.25 & 0.75 & 0.25 \\ \frac{1}{12} & 0.25 & \frac{5}{12} \end{pmatrix}$.