Answer on Question #59789 – Math – Differential Equations

Question

Use the annihilator method to solve

$y''' + y'' = 8x^2$

Solution

Equation $y''' + y'' = 8x^2$ is equivalent to $(D^3 + D^2)y = 8x^2$.

The homogeneous equation is

$(D^3 + D^2)y = 0,$

$D^2(D + 1)y = 0.$

Its solution is $y_h = c_1e^{-x} + c_2 + c_3x.$

D^3 annihilates x^2, D^3 annihilates $8x^2$:

$D^3(x^2) = 0, D^3(8x^2) = 0.$

Applying D^3 to both sides of

$$(D^3 + D^2)y = 8x^2$$

gives us

$$D^3(D^3 + D^2)y = D^3 8x^2 = 0,$$

$$D^5(D + 1)y = 0. \qquad (1)$$

The general solution to equation (1) is

$y = y_h + y_p = c_1e^{-x} + c_2 + c_3x + c_4x^2 + c_5x^3 + c_6x^4,$

where $y_h = c_1e^{-x} + c_2 + c_3x,\ y_p = c_4x^2 + c_5x^3 + c_6x^4.$

Putting

$y_p = c_4x^2 + c_5x^3 + c_6x^4,$

$y_p' = 2c_4x + 3c_5x^2 + 4c_6x^3,$

$y_p'' = 2c_4 + 6c_5x + 12c_6x^2,$

$y_p''' = 6c_5 + 24c_6x$

into the original differential equation $y''' + y'' = 8x^2$ gives us

$y_p''' + y_p'' = (6c_5 + 24c_6x) + (2c_4 + 6c_5x + 12c_6x^2) = (6c_5 + 2c_4) + (24c_6 + 6c_5)x + 12c_6x^2 = 8x^2,$

hence

$12c_6 = 8,$

$24c_6 + 6c_5 = 0,$

$6c_5 + 2c_4 = 0.$

Next,

$c_6 = \frac{8}{12} = \frac{2}{3}$,

$c_5 = -\frac{24c_6}{6} = -4c_6 = -4 \cdot \frac{2}{3} = -\frac{8}{3}$,

$c_4 = \frac{-6c_5}{2} = -3c_5 = -3 \cdot \left(-\frac{8}{3}\right) = 8$.

Thus, $y_p = c_4x^2 + c_5x^3 + c_6x^4 = 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$ and

$y = y_h + y_p = c_1e^{-x} + c_2 + c_3x + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$, $c_1, c_2, c_3 \in \mathbb{R}$.

Answer: $y = c_1e^{-x} + c_2 + c_3x + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$.

La respuesta al Problema #59789 – Matemática – Ecuaciones diferenciales

Problema

Resuelva

$y''' + y'' = 8x^2$

metodo anulador.

Solución

Una ecuación diferencial como $y''' + y'' = 8x^2$ se puede escribir en la forma $(D^3 + D^2)y = 8x^2$.

La ecuación homogénea es

$(D^3 + D^2)y = 0,$

$D^2(D + 1)y = 0.$

La solución de la ecuación es

$y_h = c_1e^{-x} + c_2 + c_3x.$

D^3 anula a x^2, D^3 anula a $8x^2$:

$D^3(x^2) = 0, D^3(8x^2) = 0.$

Aplicamos operador D^3 a ambos lados de

$$(D^3 + D^2)y = 8x^2$$

tenemos

$$D^3(D^3 + D^2)y = D^3 8x^2 = 0,$$

$$D^5(D + 1)y = 0. \qquad (1)$$

La solución general de la equación (1) es

$y = y_h + y_p = c_1e^{-x} + c_2 + c_3x + c_4x^2 + c_5x^3 + c_6x^4,$

donde $y_h = c_1e^{-x} + c_2 + c_3x,\ y_p = c_4x^2 + c_5x^3 + c_6x^4.$

Sustituimos

$y_p = c_4x^2 + c_5x^3 + c_6x^4,$

$y_p' = 2c_4x + 3c_5x^2 + 4c_6x^3,$

$y_p'' = 2c_4 + 6c_5x + 12c_6x^2,$

$y_p''' = 6c_5 + 24c_6x$

en la ecuación $y''' + y'' = 8x^2$ y simplificamos:

$y_p''' + y_p'' = (6c_5 + 24c_6x) + (2c_4 + 6c_5x + 12c_6x^2) = (6c_5 + 2c_4) + (24c_6 + 6c_5)x + 12c_6x^2 = 8x^2,$

Igualamos los coeficientos y obtenemos las ecuaciones

$12c_6 = 8,$

$24c_6 + 6c_5 = 0,$

$6c_5 + 2c_4 = 0,$

cuyas soluciones son

$c_6 = \frac{8}{12} = \frac{2}{3},$

$$c_5 = -\frac{24c_6}{6} = -4c_6 = -4 \cdot \frac{2}{3} = -\frac{8}{3},$$

$c_4 = \frac{-6c_5}{2} = -3c_5 = -3 \cdot \left(-\frac{8}{3}\right) = 8.$

Por lo tanto, $y_p = c_4x^2 + c_5x^3 + c_6x^4 = 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$

y la solución general de $y''' + y'' = 8x^2$ es

$y = y_h + y_p = c_1e^{-x} + c_2 + c_3x + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$, $c_1, c_2, c_3 \in \mathbb{R}$.

La respuesta al problema: $y = c_1e^{-x} + c_2 + c_3x + 8x^2 - \frac{8}{3}x^3 + \frac{2}{3}x^4$.