Answer on Question #59781 - Math – Trigonometry

Question

How to solve equations of the form 4sinAcosA=1

Solution

It is known that

2sinAcosA =sin2A;

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$;

$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$;

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$,

where $\arcsin(x)$ is the inverse sine function.

Besides, the period of the sine function is 2π, hence $\sin(x) = \sin(x + 2\pi) = \sin(x + 2\pi n)$, where n is integer.

Next,

4sinAcosA = 1;

2 ·(2sinAcosA) = 1;

2sin2A = 1;

sin2A=1/2;

$2A = \frac{\pi}{6} + 2\pi n$ and $2A = \frac{5\pi}{6} + 2\pi n$, where n is integer;

$A = \frac{\pi}{12} + \pi n$ and $A = \frac{5\pi}{12} + \pi n$, where n is integer.

More general form of solution is

$2A = (-1)^k \arcsin\left(\frac{1}{2}\right) + k\pi$, where k is integer;

$2A = (-1)^k \frac{\pi}{6} + k\pi$, where k is integer;

$A = (-1)^k \frac{\pi}{12} + \frac{k\pi}{2}$, where k is integer.

The equation 4sinAcosA=1 has only four roots in the range $[0; 2\pi]$:

$\frac{\pi}{12}$ (or 15°), $\frac{5\pi}{12}$ (or 75°) , $\frac{13\pi}{12}$ (or 195°), $\frac{17\pi}{12}$ (or 255°).

Answer: $A = (-1)^k \frac{\pi}{12} + \frac{k\pi}{2}$, where k is integer.