Answer on Question #59674 – Math – Calculus

Question

If $\varphi = 2xz^4 - x^2y$

find $|\nabla\varphi|$.

Solution

We know that

$$\nabla\varphi = \left(\frac{\partial\varphi}{\partial x}; \frac{\partial\varphi}{\partial y}; \frac{\partial\varphi}{\partial z}\right)$$

and

$$|\nabla\varphi| = \sqrt{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2}.$$

If $\varphi = 2xz^4 - x^2y$ then we have

$$\frac{\partial\varphi}{\partial x} = 2z^4 - 2xy;$$
$$\frac{\partial\varphi}{\partial y} = -x^2;$$
$$\frac{\partial\varphi}{\partial z} = 8xz^3;$$

and finally

$$|\nabla\varphi| = \sqrt{(2z^4 - 2xy)^2 + (-x^2)^2 + (8xz^3)^2} =$$

$$= \sqrt{4z^8 - 8xyz^4 + 4x^2y^2 + x^4 + 64\boldsymbol{x^2}z^6} = \sqrt{4z^8 + 64\boldsymbol{x^2}z^6 - 8xyz^4 + x^4 + 4x^2y^2}.$$

Answer: $|\nabla\varphi| = \sqrt{4z^8 + 64\boldsymbol{x^2}z^6 - 8xyz^4 + x^4 + 4x^2y^2}$.