Answer on Question #59580 – Math – Calculus

Question

6. If A=xz^3i−2x^2yzj+2yz^4, find ∇×A at point (1,-1,1).

(a) 2j+3k

(b) 2i+j74k

(c) i+3j+5k

(d) 3j+4k

Solution

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = i\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) - j\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + k\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) =$$

$$= i(2z^4 + 2x^2y) - j(-3xz^2) + k(-4xyz) = 2i(z^4 + x^2y) + 3xz^2j - 4xyzk$$

$$\nabla \times A(1,-1,1) = 2i(1-1) + 3j + 4k = 3j + 4k = (0,3,4)$$

Answer: (d) 3j+4k.

Question

7. Given that A=A1i+A2j+A3k and r=xi+yj+zk , evaluate ∇ · (A×r) if ∇×A=0

(a) 0

(b) 3

(c) 2

(d) 5

Solution

$$A \times r = \begin{vmatrix} i & j & k \\ x & y & z \\ A_1 & A_2 & A_3 \end{vmatrix} = i(yA_3 - zA_2) - j(xA_3 - zA_1) + k(xA_2 - yA_1)$$

$$\nabla \cdot (\mathrm{A} \times \mathrm{r}) = \frac{\partial (A \times r)_x}{\partial x} + \frac{\partial (A \times r)_y}{\partial y} + \frac{\partial (A \times r)_z}{\partial z} =$$

$$= \frac{\partial (yA_3 - zA_2)}{\partial x} + \frac{\partial (-xA_3 + zA_1)}{\partial y} + \frac{\partial (xA_2 - yA_1)}{\partial z}$$

$$\nabla\times A=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_1 & A_2 & A_3\end{vmatrix}=i\left(\frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z}\right)-j\left(\frac{\partial A_3}{\partial x}-\frac{\partial A_1}{\partial z}\right)+k\left(\frac{\partial A_2}{\partial x}-\frac{\partial A_1}{\partial y}\right)=0$$

$$\frac{\partial(yA_3-zA_2)}{\partial x}+\frac{\partial(-xA_3+zA_1)}{\partial y}+\frac{\partial(xA_2-yA_1)}{\partial z}=$$
$$=y\frac{\partial A_3}{\partial x}-y\frac{\partial A_1}{\partial z}-z\frac{\partial A_2}{\partial x}+z\frac{\partial A_1}{\partial y}-x\frac{\partial A_3}{\partial y}+x\frac{\partial A_2}{\partial z}=0$$

Answer: (a) 0.

Question

8. Let A=x^2yi−2xzj+2yzk, find Curl curl A.

(a) 3j+4k

(b) 2x+2)k

(c) (2x+2)j

(d) 3j−4k

Solution

$$curl\ A=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}=i\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)-j\left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right)+k\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)=$$
$$=i(2z+2x)-j(0)+k(-2z-x^2)$$

$$curl\ curl\ A=\begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (curlA)_x & (curlA)_y & (curlA)_z\end{vmatrix}=$$
$$=i\left(\frac{\partial(curlA)_z}{\partial y}-\frac{\partial A(curlA)_y}{\partial z}\right)-j\left(\frac{\partial(curlA)_z}{\partial x}-\frac{\partial(curlA)_x}{\partial z}\right)+k\left(\frac{\partial(curlA)_y}{\partial x}-\frac{\partial(curlA)_x}{\partial y}\right)$$

$$curl\ curl\ A=i(0-0)-j(-2x-2)+k(0-0)=$$

$$=(2x+2)j$$

Answer: (c) (2x+2)j**.**

Question

9. Given A=2x^2i−3yzj+xz^2k and φ=2z−x^3y, find A·∇φ at point (1,-1,1).

(a) 5

(b) 3

(c) 4

(d) 1

Solution

$$\nabla\varphi = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} = -3x^2yi - x^3j + 2k$$
$$A\nabla\varphi = -6x^4y + 3x^3yz + 2xz^2$$

A·∇φ at point (1,-1,1) is

$$A\nabla\varphi(1,-1,1) = -5(1)^4(-1) + 3(1)^3(-1)(1) + 2(1)(1)^2 = 4$$

Answer: (c) 4.

Question

10. Find the directional derivative of φ=x^2yz+4xz^2 at (1,-2,-1) in the direction 2i–j–2k

(a) 37/3

(b) 35/3

(c) 25/3

(d) 11/3

Solution

Let $\vec{l} = 2\vec{\imath} - \vec{\jmath} - 2\vec{k} \rightarrow |\vec{l}| = \sqrt{4+1+4} = 3;$

$A = (1,-2,-1)$

So, the direction cosines are

$$\cos\alpha = \frac{(\vec{l})_x}{|\vec{l}|} = \frac{2}{3}, \cos\beta = \frac{(\vec{l})_y}{|\vec{l}|} = -\frac{1}{3}, \cos\gamma = \frac{(\vec{l})_z}{|\vec{l}|} = -\frac{2}{3}.$$

Next,

$$\frac{\partial\varphi}{\partial x}(A) = \frac{\partial(x^2yz+4xz^2)}{\partial x}(A) = (2xyz + 4z^2)(A) = 2\cdot 1\cdot(-2)\cdot(-1) + 4\cdot(-1)^2 = 8,$$

$$\frac{\partial\varphi}{\partial y}(A) = \frac{\partial(x^2yz+4xz^2)}{\partial y}(A) = (x^2z)(A) = 1^2\cdot(-1) = -1,$$

$$\frac{\partial\varphi}{\partial z}(A) = \frac{\partial(x^2yz+4xz^2)}{\partial z}(A) = (x^2y + 8xz)(A) = 1^2\cdot(-2) + 8\cdot 1\cdot(-1) = -10.$$

The directional derivative of φ=x^2yz+4xz^2 at (1,-2,-1) in the direction 2i–j–2k is

$$\frac{\partial\varphi}{\partial l}(A) = \frac{\partial\varphi}{\partial x}(A)\cdot\cos\alpha + \frac{\partial\varphi}{\partial y}(A)\cdot\cos\beta + \frac{\partial\varphi}{\partial z}(A)\cdot\cos\gamma = 8\cdot\frac{2}{3} + (-1)\cdot\left(-\frac{1}{3}\right) + (-10)\cdot\left(-\frac{2}{3}\right) = \frac{37}{3}.$$

Answer: (a) 37/3.