Answer on Question #59579 - Math - Calculus

Question

1. If φ=2xz4−x2y, find |∇φ|

(a) √(93)

(b) √(80)

(c) √(12)

(d) √(110)

Solution

$$\nabla\phi = \frac{\partial\phi}{\partial x}\vec{\imath} + \frac{\partial\phi}{\partial y}\vec{\jmath} + \frac{\partial\phi}{\partial z}\vec{k} = \frac{\partial(2xz^4 - x^2y)}{\partial x}\vec{\imath} + \frac{\partial(2xz^4 - x^2y)}{\partial y}\vec{\jmath} + \frac{\partial(2xz^4 - x^2y)}{\partial z}\vec{k}$$
$$= (2z^4 - 2xy)\vec{\imath} - x^2\vec{\jmath} + 8xz^3\vec{k}$$

$$|\nabla\phi| = \sqrt{(2z^4 - 2xy)^2 + x^4 + 64x^2z^6}$$

Answer: $|\nabla\phi| = \sqrt{(2z^4 - 2xy)^2 + x^4 + 64x^2z^6}$.

Question

2. If φ(x,y,z)=3x^2y−y^3z^2, find ∇φ at point (1,-2,-1)

(a) −12i−9j−16k

(b) i−3j−k

(c) 2i−5j−6k

(d) −3i−4j−2k

Solution

$$\nabla\phi = \frac{\partial\phi}{\partial x}\vec{\imath} + \frac{\partial\phi}{\partial y}\vec{\jmath} + \frac{\partial\phi}{\partial z}\vec{k} = \frac{\partial(3x^2y - y^3z^2)}{\partial x}\vec{\imath} + \frac{\partial(3x^2y - y^3z^2)}{\partial y}\vec{\jmath} + \frac{\partial(3x^2y - y^3z^2)}{\partial z}\vec{k} =$$
$$= 6xy\vec{\imath} + (3x^2 - 3y^2z^2)\vec{\jmath} - 2zy^3\vec{k}$$

Then we put point's coordinates in the previous expression:

$$\nabla\phi(1;-2;-1) = 6\cdot 1\cdot(-2)\cdot\vec{\imath} + (3\cdot 1^2 - 3\cdot(-2)^2\cdot(-1)^2)\vec{\jmath} - 2\cdot(-1)\cdot(-2)^3\vec{k} =$$
$$= -12\vec{\imath} - 9\vec{\jmath} - 16\vec{k}$$

Answer: (a) −12i−9j−16k.

Question

3. Find a unit normal to the surface x^2y+2xz=4 at point (2,-2,3):

(a) 2/3i−2/3j−2/3k

(b) −1/5i+2/5j+2/5k

(c) −1/3i+2/3j+2/3k

(d) −1/7i+2/7j+2/7k

Solution

First, we rewrite the surface equation in the form of $F(x, y, z) = 0$.

$$x^2y + 2xz - 4 = 0.$$

A normal to a surface can be found as $\vec{F}'(A) = (F_x{}'(A); F_y{}'(A); F_z{}'(A))$, where A is the point (2;-2;3).

$$F_x{}'(A) = 2xy + 2z = -2,$$

$$F_y{}'(A) = x^2 = 4,$$

$$F_z{}'(A) = 2x = 4,$$

$$\left|\vec{F'}(A)\right| = \sqrt{(-2)^2 + 4^2 + 4^2} = 6.$$

A unit normal vector is

$$\vec{n} = \frac{\vec{F}'(A)}{|\vec{F}'(A)|} = \frac{-2\vec{\imath}+4\vec{\jmath}+4\vec{k}}{6} = -\frac{2}{6}\vec{\imath} + \frac{4}{6}\vec{\jmath} + \frac{4}{6}\vec{k} = -\frac{1}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} + \frac{2}{3}\vec{k}.$$

Answer: (c) −1/3i+2/3j+2/3k.

Question

4. Let φ(x,y,z)=xy^2z and A=xzi−xy^2j+yz^2k,find ∂^3/∂x^2∂z(φA)

(a) 2i+2j−5k

(b) 5i−k

(c) 4i−2j

(d) i+j

Solution

$$(\phi A) = xy^2z \cdot (xz; -xy^2; yz^2) = (x^2y^2z^2; -x^2y^4z; xy^3z^3)$$

$$\frac{\partial^3}{\partial x^2 \partial z}(x^2y^2z^2) = \frac{\partial^2}{\partial x^2}\left(\frac{\partial}{\partial z}(x^2y^2z^2)\right) = \frac{\partial^2}{\partial x^2}(2x^2y^2z) = 4y^2z$$

$$\frac{\partial^3}{\partial x^2 \partial z}(-x^2y^4z) = \frac{\partial^2}{\partial x^2}(-x^2y^4) = -2y^4$$

$$\frac{\partial^3}{\partial x^2 \partial z}(xy^3z^3) = \frac{\partial^2}{\partial x^2}(3xy^3z^2) = 0.$$

Thus,

$$\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A}) = 4y^2z\,\vec{\imath} - 2y^4\vec{\jmath}.$$

If we need answer with only numbers we need to choose the point where we calculate $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$.

Also we can see that answers **2i+2j−5k** and **5i−k** are false, because k-th component of the vector $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ is zero. And the answer **i+j** is false because j-th component of the vector $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ is always less or equal than zero:

$$-2y^4 \le 0.$$

So, only **4i−2j** can be answer to the problem.

Answer: (c) 4i−2j.

Question

5. Given that φ=2x^2y−xz^3 find ∇^2φ

(a) 2y−6xz

(b) 4y−6xz

(c) 2y−xz

(d) y+6xz

Solution

$$\nabla\phi = \frac{\partial\phi}{\partial x}\vec{\imath} + \frac{\partial\phi}{\partial y}\vec{\jmath} + \frac{\partial\phi}{\partial z}\vec{k} = \frac{\partial(2x^2y - xz^3)}{\partial x}\vec{\imath} + \frac{\partial(2x^2y - xz^3)}{\partial y}\vec{\jmath} + \frac{\partial(2x^2y - xz^3)}{\partial z}\vec{k} =$$

$$= (4xy - z^3)\vec{\imath} + 2x^2\vec{\jmath} - 3xz^2\vec{k},$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial^2(2x^2y - xz^3)}{\partial x^2} + \frac{\partial^2(2x^2y - xz^3)}{\partial y^2} + \frac{\partial^2(2x^2y - xz^3)}{\partial z^2} =$$

$$= \frac{\partial}{\partial x}\frac{\partial}{\partial x}(2x^2y - xz^3) + \frac{\partial}{\partial y}\frac{\partial}{\partial y}(2x^2y - xz^3) + \frac{\partial}{\partial z}\frac{\partial}{\partial z}(2x^2y - xz^3) =$$

$$= \frac{\partial}{\partial x}(4xy - z^3) + \frac{\partial}{\partial y}(2x^2) + \frac{\partial}{\partial z}(-3xz^2) = 4y + 0 + (-6xz) = 4y - 6xz.$$

Answer: (b) 4y−6xz.