Answer on Question #59574 – Math – Calculus

Question

5. If A=5t^2+tj−t^3k and B=sinti−costj. evaluate d/dt(A·B)
(a) (5t^2−1)cost+11tsint
(b) (5t−1)sint+11tcost
(c) −1cost+2tsint
(d) (5t^2−1)sint+11tcost

Solution

$$A = 5t^2\boldsymbol{i} + t\boldsymbol{j} - t^3\boldsymbol{k}, \quad B = sint\boldsymbol{i} - cost\boldsymbol{j},$$

$$AB = 5t^2 sint - tcost,$$

$$\frac{d}{dt}(AB) = \frac{d}{dt}(5t^2 sint - tcost) = 10tsint + 5t^2 cost - cost + tsint =$$

$$= (5t^2 - 1)cost + 11tsint.$$

Answer: a) (5t^2−1)cost+11tsint

Question

6 If A=5t^2+tj−t^3k and B=sinti−costj . evaluate d/dt(A×B)
(a) (t^3sint−3t^2cost)i−(t^3cost−3t^2sint)j+(5t^2sint−11tcost−sint)k
(b) (t^2sint−3tcost)i−(t^3cost−3tsint)j+(5sint−11tcost−sint)k
(c) (tsint−3t^2cost)i−(t^3cost−3t^2sint)j+(5t^2cost−11tcost−cost)k
(d) (tcost−3t^2cost)i−(t^3sint−3t^2cost)j+(5t^2cost−11tsint−cost)k

Solution

$$A \times B = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 5t^2 & t & -t^3 \\ sint & -cost & 0 \end{vmatrix} = -t^3 cost\boldsymbol{i} - t^3 sint\boldsymbol{j} - (5t^2 cost + tsint)\boldsymbol{k}$$

$$\frac{d}{dt}(A \times B) = \frac{d(-t^3 cost)}{dt}\boldsymbol{i} + \frac{d(-t^3 sint)}{dt}\boldsymbol{j} + \frac{d(-5t^2 cost - tsint)}{dt}\boldsymbol{k} = (t^3 sint -$$
$$3t^2 cost)\boldsymbol{i} - (t^3 cost + 3t^2 sint)\boldsymbol{j} +$$

$+(5t^2 sint - 11tcost - sint)\boldsymbol{k}.$

Answer: (t^3sint−3t^2cost)i−(t^3cost+3t^2sint)j+(5t^2sint−11tcost−sint)k

Question

9. Let A=x^2yzi−2xz^3j−xz^2 and B=4zi+yj+4x^2k , find ∂^2/∂x∂y(A×B) at (1,0,-2)
(a) 2i−8j
(b) −4i−8j
(c) −i−3j
(d) 5i−2j

Solution

$A = x^2yz\boldsymbol{i} - 2xz^3\boldsymbol{j} - xz^2\boldsymbol{k},\ \ B = 4z\boldsymbol{i} + y\boldsymbol{j} + 4x^2\boldsymbol{k},$

$$A \times B = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x^2yz & -2xz^3 & -xz^2 \\ 4z & y & 4x^2 \end{vmatrix} = (xyz^2 - 8x^3z^3)\boldsymbol{i} - 4(x^4yz + xz^3)\boldsymbol{j} +$$

$$+(x^2y^2z + 8xz^4)\boldsymbol{k}.$$

$$\frac{\partial^2}{\partial x \partial y}(A \times B) = z^2\ \boldsymbol{i} - 16x^3z\boldsymbol{j} + 4xyz\boldsymbol{k}.$$

$$\frac{\partial^2}{\partial x \partial y}(A \times B)|_{(1,0,-2)} = 4i + 32j.$$

Answer: $4i + 32j$**.**