Answer on Question #59573 – Math – Calculus

Question

1. Given that $\boldsymbol{A} = \sin(t)\,\boldsymbol{i} + \cos(t)\,\boldsymbol{j} + t\,\boldsymbol{k}$, evaluate $\left|\frac{d^2\boldsymbol{A}}{dt^2}\right|$.

(a) 4
(b) 1
(c) 3
(d) 2

Solution

First of all, let us calculate $\frac{d\boldsymbol{A}}{dt}$:

$$\frac{d\boldsymbol{A}}{dt} = \frac{d}{dt}(sin(t))\boldsymbol{i} + \frac{d}{dt}(cos(t))\boldsymbol{j} + \frac{d}{dt}(cos(t))\,\boldsymbol{k} = \cos(t)\,\boldsymbol{i} - \sin(t)\,\boldsymbol{j} + 1\cdot\boldsymbol{k}.$$

For the second derivative we have:

$$\frac{d^2\boldsymbol{A}}{dt^2} = \frac{d}{dt}(cos(t))\boldsymbol{i} + \frac{d}{dt}(-sin(t))\boldsymbol{j} + \frac{d}{dt}(1)\boldsymbol{k} = -\sin(t)\,\boldsymbol{i} - \cos(t)\,\boldsymbol{j}.$$

Therefore, the absolute values of the last vector is

$$\left|\frac{d^2\boldsymbol{A}}{dt^2}\right| = \sqrt{\sin^2(t) + \cos^2(t)} = 1.$$

Answer: (b) $\left|\frac{d^2\boldsymbol{A}}{dt^2}\right| = 1.$

Question

2. A particle moves along the curve $x(t) = 2t^2, y(t) = t^2 - 4t$ and $z(t) = 3t - 5$, where t is the time. Find the components of the velocity at $t = 1$ in the direction $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$.

(a) 8√(14)/7

(b) −2√(14)/7

(c) 3√(14)/7

(d) −5√(14)/7

Solution

The position vector of the particle is given by

$$\boldsymbol{r}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k}$$

Therefore, we can find the velocity of the particle as the first time derivative of the last expression:

$$\boldsymbol{v}(t) = \frac{d\boldsymbol{r}}{dt} = \frac{dx(t)}{dt}\boldsymbol{i} + \frac{dy(t)}{dt}\boldsymbol{j} + \frac{dz(t)}{dt}\boldsymbol{k} = \frac{d(2t^2)}{dt}\boldsymbol{i} + \frac{d(t^2 - 4t)}{dt}\boldsymbol{j} + \frac{d(3t - 5)}{dt}\boldsymbol{k}$$
$$= 4t\,\boldsymbol{i} + (2t - 4)\boldsymbol{j} + 3\boldsymbol{k}$$

At the time $t = 1$ it is equal to

$$\boldsymbol{v}(1) = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$$

Now, let us consider the direction vector

$$\boldsymbol{n} = \boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$$

The component of the velocity at $t = 1$ in the direction $\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k}$ is equal to

$$\frac{(\boldsymbol{v}\cdot\boldsymbol{n})}{|\boldsymbol{n}|}=\frac{4\cdot 1+(-2)\cdot(-3)+3\cdot 2}{\sqrt{1+(-3)\cdot(-3)+2\cdot 2}}=\frac{16}{\sqrt{14}}=\frac{16\sqrt{14}}{14}=\frac{8\sqrt{14}}{7}$$

Answer: (a) $\frac{8\sqrt{14}}{7}$.