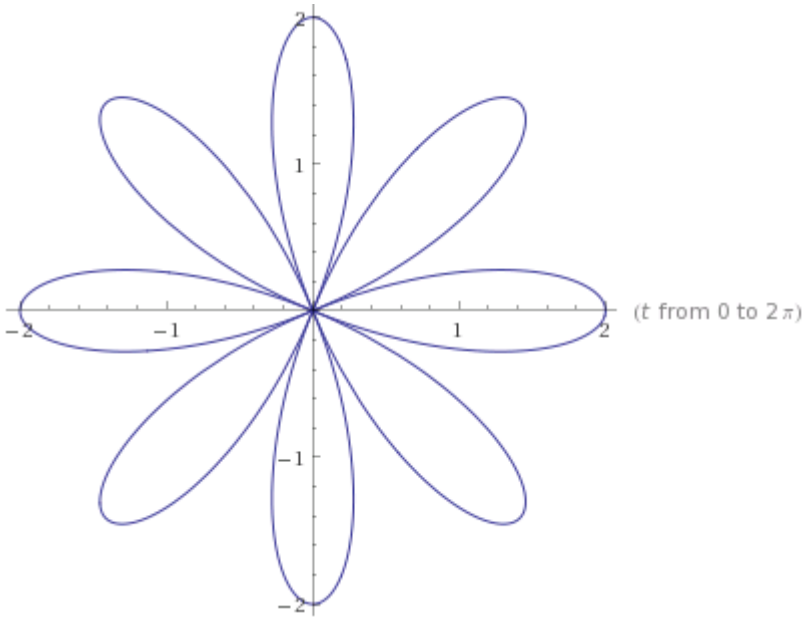
Answer on Question #59528 – Math – Calculus

Question

Determine the area under the curve $s = 2\cos 4\theta$ in the range $\theta = 0$ to $\pi/4$ radians.

Solution

Plot of $s = 2cos\ 4\theta$ is



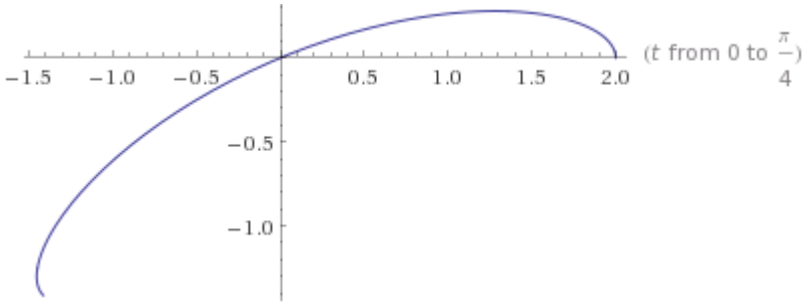
In the polar coordinates we have

$s(0) = 2\cos(4 \cdot 0) = 2,$

$s\left(\frac{\pi}{8}\right) = 2\cos\left(4 \cdot \frac{\pi}{8}\right) = 2\cos\left(\frac{\pi}{2}\right) = 0,$

$s\left(\frac{\pi}{4}\right) = 2\cos\left(4 \cdot \frac{\pi}{4}\right) = 2\cos(\pi) = -2,$

Plot of $s = 2cos\ 4\theta$, $0 \le \theta \le \frac{\pi}{4}$ is



The area under the curve $s = 2\cos 4\theta$ in the range $\theta = 0$ to $\pi/4$ radians:

$$A = \frac{1}{2}\int_0^{\pi/4} s^2(\theta)\, d\theta = \frac{1}{2}\int_0^{\pi/4} 4(\cos 4\theta)^2\, d\theta =$$

$$= 2\int_0^{\pi/4} \frac{1+\cos 8\theta}{2}\, d\theta = \int_0^{\pi/4} (1+\cos 8\theta) d\theta = \theta + \frac{\sin 8\theta}{8}\Big|_0^{\pi/4} =$$

$$= \frac{\pi}{4} + \frac{sin\left(8\cdot\frac{\pi}{4}\right)}{8} - \left(0 + \frac{sin(8\cdot 0)}{8}\right) =$$

$$= \frac{\pi}{4} \approx 0.78540.$$

Answer. $\frac{\pi}{4}$.