Answer on Question #59505 – Math – Calculus

Question

The Voltage, v across the plates of the charging capacitor varies with time, t according to the formula

v = V (1 - e^-(t/T)),

where T = CR and is called the time constant c = 100nF, R = 47kOhms and V = 5V.

Differentiate the charging equation and find the rate of change of voltage at 6ms.

Solution

The charging equation is

$$v(t) = V\left(1 - e^{-\frac{t}{T}}\right),$$

Differentiating the charging equation

$$\dot{v}(t) = \frac{dv}{dt} = \frac{V}{T}e^{-\frac{t}{T}},$$

where

C = 100nF=10^{-7}F, R = 47kOhms=$47\cdot10^{3}$Ohms,

$T = CR = 10^{-7}\text{F}\cdot 47\cdot 10^{3}\text{Ohms} = 47\cdot 10^{-4}\text{F}\cdot\text{Ohms} = \ 4.7\ ms.$

The rate of change of voltage at 6ms is

$$\dot{v}(6\ ms) = \frac{dv}{dt}(6\ ms) = \frac{5V}{4.7ms}e^{-\frac{6}{4.7}} = 0.2968\frac{V}{ms}.$$

Answer: $\frac{dv}{dt} = \frac{V}{T}e^{-\frac{t}{T}};\ \ \frac{dv}{dt}(6\ ms) = 0.2968\frac{V}{ms}.$