Answer on Question #59454 – Math – Differential Equations

Question

8. One of these is a homogeneous equation

I. h(x,y)=x^3+2xy+3xy^2+4y^3

II. h(x,y)=x^2+2x^2y+3xy^2+4y^3

III. h(x,y)=x^3+2x^2y+3xy^2+4y^3

IV. h(x,y)=x^3+2xy+3xy^2+4y^2

Solution

The equation $M(x,y)dx+N(x,y)dy=0$ is a homogeneous type if M(x, y) and N(x, y) are homogeneous functions of the same degree *n* and homogeneous function of the degree *n* is a function which satisfies the condition $f(tx,ty)=t^n f(x,y)$.
Check that the function III. h(x,y)=x^3+2x^2y+3xy^2+4y^3
is homogeneous of order 3:
h(tx,ty)=(tx)^3+2(tx)^2ty+3tx(ty)^2+4(ty)^3= t^3x^3+2t^3x^2y+3y^3xy^2+4t^3y^3=
=t^3(x^3+2x^2y+3xy^2+4y^3)=t^3h(x,y).
Answer: III. h(x,y)=x^3+2x^2y+3xy^2+4y^3.

Question

9. One of the following is not a separable equation

i. dy/dx=e^x+y

II. dy/dx=e^xy

III. dy/dx=x^2(y+y^2)

IV. (1+y^2)dx+(1+x^3)dy

Solution

The differential equation of the form $\frac{dy}{dx}=f(x,y)$ is called separable, if *f*(*x*,*y*) = *h*(*x*) *g*(*y*),
Consider i. dy/dx=e^x+y, f(x,y) =e^x+y =h(x)+g(y) is the sum of functions, so this equation is not a separable equation.

Answer: i. dy/dx=e^x+y.