Answer on Question #59424 – Math – Calculus

Question

Integrate by partial fraction $\frac{5x+2}{3x^2+x+4}$.

Solution

Method 1

$3x^2 + x + 4 = 0,$

$D = 1 - 4 \cdot 3 \cdot 4 < 0$, hence $3x^2 + x + 4$ does not have real roots.

$$\int \frac{(5x+2)dx}{3x^2+x+4} = 5\int \frac{xdx}{3x^2+x+4} + 2\int \frac{dx}{3x^2+x+4} = \frac{5}{6}\int \frac{(6x+1)dx}{3x^2+x+4} + \left(2 - \frac{5}{6}\right)\int \frac{dx}{3x^2+x+4} = \frac{5}{6}\int \frac{d(3x^2+x+4)}{3x^2+x+4} +$$

$$+\frac{7}{6}\int \frac{dx}{3\left(x^2+\frac{x}{3}+\frac{4}{3}\right)} = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{6\cdot 3}\int \frac{dx}{x^2+2\frac{x}{6}+\frac{1}{36}+\frac{4}{3}-\frac{1}{36}} = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{18}\int \frac{dx}{\left(x+\frac{1}{6}\right)^2+\frac{47}{36}} =$$

$$= \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{18}\int \frac{dx}{\left(x+\frac{1}{6}\right)^2+\left(\frac{\sqrt{47}}{6}\right)^2} = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{\frac{7}{18}}{\frac{\sqrt{47}}{6}}\arctan\frac{x+\frac{1}{6}}{\frac{\sqrt{47}}{6}} + C_1 =$$

$$= \frac{5}{6}\ln|3x^2 + x + 4| + + \frac{7\cdot 6}{18\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C_1 = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C_1,$$

where C_1 is an integration constant.

Method 2

$3x^2 + x + 4 = 0,$

$D = 1 - 4 \cdot 3 \cdot 4 < 0$, hence $3x^2 + x + 4$ does not have real roots.

$$\int \frac{(5x+2)dx}{3x^2+x+4} = \int \frac{(5x+2)dx}{3\left(x^2+\frac{x}{3}+\frac{4}{3}\right)} = \frac{1}{3}\int \frac{(5x+2)dx}{x^2+2\frac{x}{6}+\frac{1}{36}+\frac{4}{3}-\frac{1}{36}} = \frac{1}{3}\int \frac{(5x+2)dx}{\left(x+\frac{1}{6}\right)^2+\frac{47}{36}} = \frac{1}{3}\int \frac{\left(5x+\frac{5}{6}\right)dx}{\left(x+\frac{1}{6}\right)^2+\frac{47}{36}} + \frac{1}{3}\left(2 - \frac{5}{6}\right)\int \frac{dx}{\left(x+\frac{1}{6}\right)^2+\frac{47}{36}} =$$

$$= \frac{5}{3\cdot 2}\int \frac{2\left(x+\frac{1}{6}\right)dx}{\left(x+\frac{1}{6}\right)^2+\frac{47}{36}} + \frac{7}{3\cdot 6}\int \frac{dx}{\left(x+\frac{1}{6}\right)^2+\left(\frac{\sqrt{47}}{6}\right)^2} = \frac{5}{3\cdot 2}\ln\left|\left(x+\frac{1}{6}\right)^2 + \frac{47}{36}\right| + \frac{7}{3\cdot 6}\cdot\frac{1}{\frac{\sqrt{47}}{6}}\arctan\frac{x+\frac{1}{6}}{\frac{\sqrt{47}}{6}} + C_2 =$$

$$= \frac{5}{6}\ln\left|x^2 + \frac{x}{3} + \frac{4}{3}\right| + \frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C_2 = \frac{5}{6}\ln\left|x^2 + \frac{x}{3} + \frac{4}{3}\right| + \frac{5}{6}\ln(3) + \frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} +$$

$$+C_2 - \frac{5}{6}\ln(3) = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C_2 - \frac{5}{6}\ln(3) = \frac{5}{6}\ln|3x^2 + x + 4| +$$

$$+\frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C_1,$$

where $C_1 = C_2 - \frac{5}{6}\ln(3)$ is an integration constant.

Answer: $\int \frac{(5x+2)dx}{3x^2+x+4} = \frac{5}{6}\ln|3x^2 + x + 4| + \frac{7}{3\sqrt{47}}\arctan\frac{6x+1}{\sqrt{47}} + C.$