

Answer on Question #59340 – Math – Trigonometry

Question

The value $\frac{\pi}{24}$ is a solution for the equation $4\cos^2(4x) - 3 = 0$.

False

True

Solution

$$4\cos^2\left(4 \cdot \frac{\pi}{24}\right) - 3 = 4\cos^2\left(\frac{\pi}{6}\right) - 3 = 4\left(\frac{\sqrt{3}}{2}\right)^2 - 3 = 3 - 3 = 0.$$

Answer: True.

Question

Solve on the interval $[0, 2\pi)$:

$$(\cos x + 1)(2\cos^2 x - 3\cos x - 2) = 0$$

$$x = 2\pi, x = \frac{\pi}{2}, x = \frac{\pi}{3}$$

$$x = \frac{\pi}{6}, x = \frac{7\pi}{6}$$

$$x = \pi, x = \frac{2\pi}{3}, x = \frac{4\pi}{3}$$

$$x = 2\pi, x = \frac{\pi}{2}, x = \frac{5\pi}{4}$$

Solution

$$(\cos x + 1)(2\cos^2 x - 3\cos x - 2) = 0, 0 \leq x < 2\pi;$$

$$\cos x + 1 = 0, 0 \leq x < 2\pi, \text{ or } 2\cos^2 x - 3\cos x - 2 = 0, 0 \leq x < 2\pi;$$

$$\cos x + 1 = 0, 0 \leq x < 2\pi;$$

$$\cos x = -1, \quad 0 \leq x < 2\pi;$$

$$x = \pi;$$

$$2\cos^2 x - 3\cos x - 2 = 0, \quad 0 \leq x < 2\pi;$$

$$D = (-3)^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25;$$

$$\cos x = \frac{3+5}{2 \cdot 2} = \frac{8}{4} = 2 \text{ or } \cos x = \frac{3-5}{2} = -1, \quad 0 \leq x < 2\pi;$$

$\cos x = 2$ does not have real solutions on $0 \leq x < 2\pi$;

$\cos x = -\frac{1}{2}$ has solutions $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$ on $0 \leq x < 2\pi$.

Answer: $x = \frac{2\pi}{3}, x = \frac{4\pi}{3}$.

Question

Evaluate $\cot\left(\cos^{-1}\left(-\frac{15}{17}\right)\right)$. Enter your answer as a fraction using the slash bar (/). _____

Solution

$$\cot(\cos^{-1}(x)) = x/\sqrt{1-x^2};$$

$$\cot\left(\cos^{-1}\left(-\frac{15}{17}\right)\right) = -\frac{15}{17}/\sqrt{1-\left(-\frac{15}{17}\right)^2} = -\frac{15}{17}/\frac{8}{17} = -\frac{15}{8} = -1.875.$$

Answer: $-15/8$.