Answer on Question #59281 – Math – Analytic Geometry

Question

a circle is tangent to lines 5x+2y-10=0 and 5x+2y+2=0. find its area and center.

Solution

Method 1

These straight lines are parallel, because their slopes are equal, so there can be the infinite number of circles.

Take a point $(x_A; y_A) = (2; 0)$ which lies on the straight line 5x+2y-10=0.

The distance between straight lines 5x+2y-10=0 and 5x+2y+2=0 is equal to the distance between the point $(x_A; y_A) = (2; 0)$ and the straight line 5x+2y+2=0 :

$$d = \frac{|5 \cdot 2 + 2 \cdot 0 + 2|}{\sqrt{5^2 + 2^2}} = \frac{12}{\sqrt{29}}.$$

The length of circle's radius is equal to

$$r = \frac{d}{2} = \frac{12}{2\sqrt{29}} = \frac{6}{\sqrt{29}}.$$

The area of the circle is equal to

$$S = \pi r^2 = \pi \left(\frac{6}{\sqrt{29}}\right)^2 = \frac{36\pi}{29}$$

Method 2

A circle is tangent to lines 5x + 2y - 10 = 0 and 5x + 2y + 2 = 0, so the equations of the lines are:
y = -2.5x + 5, y = -2.5x - 1.
These lines are parallel, because their slopes are equal, so there can be the infinite number of circles with the centers on the line y = -2.5x + 2.
As the slope equals -2.5, then tangent of this angle is tan(a) = -2.5, where a is an angle between the line and the x-axis, so the angle is 111.8°. Let's consider the rectangular triangle between the lines

y = -2.5x - 1 and y = -2.5x + 5, one its cathetus is the diameter of the circle, the hypotenuse equals 6 (distance between two lines, which is parallel to the y-axis, for example, distance between points (0;-1) and (0;5) which lie on the lines given), its angle between cathetus, which is the diameter of the circle, and hypotenuse equals to the smaller angle between the line y = -2.5x + 5 and x-axis, because our rectangular triangle is similar to another rectangular triangle between the line

y = -2.5 + 5, x-axis and y-axis (3 equal angles), so that angle equals 180° - a, then

$\frac{2r}{6} = cos(180^{\circ} - a)$ and the length of the radius of the circle is

r = 6cos(180° - a)/2 = 3cos(68.2°) =$3 \cdot 0.371$ ≈ 1.113
The area of the circle is S = π·r^2 = π·1.113^2 ≈ 3.89.